

Modern Algebra

Previous year Questions
from 2016 to 1992

Ramanasri

2017

2016

1. Let K be a field and $K[X]$ be the ring of polynomials over K in a single variable X for a polynomial $f \in K[X]$. Let (f) denote the ideal in $K[X]$ generated by f . Show that (f) is a maximal ideal in $K[X]$ if and only if f is an irreducible polynomial over K . **(10 marks)**
2. Let p be a prime number and Z_p denote the additive group of integers modulo p . Show that every non-zero element Z_p generates Z_p . **(15 marks)**
3. Let K be an extension of a field F . Prove that the elements of K which are algebraic over F form a subfield of K . Further, if $F \subset K \subset L$ are fields, L is algebraic over K and K is algebraic over F , then prove that L is algebraic over F . **(20 marks)**
4. Show that every algebraically closed field is infinite. **(15 marks)**

2015

5. (i) How many generators are there of the cyclic group G of order 8? Explain. **(5 Marks)**
(ii) Taking a group $\{e, a, b, c\}$ of order 4, where e is the identity, construct composition tables showing that one is cyclic while the other is not. **(5 Marks)**
6. Give an example of a ring having identity but a subring of this having a different identity. **(10 Marks)**
7. If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R' , prove that $\phi(1)$ is the unit element of R' . **(15 Marks)**
8. Do the following sets form integral domains with respect to ordinary addition and multiplication? If so, state if they are fields: **(5+6+4=15 Marks)**
(i) The set of numbers of the form $b\sqrt{2}$ with b rational.
(ii) The set of even integers.
(iii) The set of positive integers.

2014

9. Let G be the set of all real 2×2 $\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$ where $xz \neq 0$ matrices. Show that G is a group under matrix multiplication. Let N denote the subset $\left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R \right\}$. Is N a normal subgroup of G ? Justify your answer. **(10 Marks)**
10. Show that Z_7 is a field. Then find $([5] + [6])^{-1}$ and $(-[4])^{-1}$ in Z_7 . **(15 Marks)**
11. Show that the set $\{a + b\omega : \omega^3 = 1\}$, where a and b are real numbers, is a field with respect to usual addition and multiplication. **(15 Marks)**
12. Prove that the set $Q(\sqrt{5}) = \{a + b\sqrt{5} : a, b \in Q\}$ is a commutative ring with identity. **(15 Marks)**

2013

13. Show that the set of matrices $S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \mid a, b \in R \right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f: C \rightarrow S$ defined by $f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here R is the set of real numbers and C is the set of complex numbers) **(10 Marks)**
14. Give an example of an infinite group in which every element has finite order **(10 Marks)**
15. What are the orders of the following permutation in S_{10} ? $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$ and $(1 \ 2 \ 3 \ 4 \ 5)(6 \ 7)$ **(10 Marks)**
16. What is the maximal possible order of an element in S_{10} ? Why? Give an example of such an element. How many elements will there be in S_{10} of that order? **(13 Marks)**
17. Let $J = \{a + ib \mid a, b \in Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J : Euclidean domain, principal ideal domain, and unique factorization domain? Justify your answer **(15 Marks)**
18. Let $R^C =$ ring of all real value continuous functions on $[0, 1]$, under the operations $(f + g)x = f(x) + g(x)$, $(fg)x = f(x)g(x)$. Let $M = \left\{ f \in R^C \mid f\left(\frac{1}{2}\right) = 0 \right\}$. Is M a maximal ideal of R ? Justify your answer. **(15 Marks)**

2012

19. How many elements of order 2 are there in the group of order 16 generated by a and b such that the order of a is 8, the order of b is 2 and $bab^{-1} = a^{-1}$. **(12 Marks)**
20. How many conjugacy classes does the permutation group S_5 of permutation 5 numbers have? Write down one element in each class (preferably in terms of cycles). **(15 Marks)**
21. Is the ideal generated by 2 and X in the polynomial ring $Z[X]$ of polynomials in a single variable X with coefficients in the ring of integers Z , a principal ideal? Justify your answer **(15 Marks)**
22. Describe the maximal ideals in the ring of Gaussian integers $Z[i] = \{a + ib \mid a, b \in Z\}$. **(20 Marks)**

2011

23. Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by $f_1(z) = z, f_2(z) = 1 - z, f_3(z) = \frac{z}{(1-z)}, f_4(z) = \frac{1}{z}, f_5(z) = \frac{1}{(1-z)}, f_6(z) = \frac{(z-1)}{z}$ is a non-abelian group of order 6 w.r.t. composition of mappings **(12 Marks)**
24. Prove that a group of Prime order is abelian. **(6 Marks)**
25. How many generators are there of the cyclic group (G, \cdot) of order 8? **(6 Marks)**

26. Give an example of a group G in which every proper subgroup is cyclic but the group itself is not cyclic **(15 Marks)**
27. Let F be the set of all real valued continuous functions defined on the closed interval $[0, 1]$. Prove that $(F, +, \cdot)$ is a Commutative Ring with unity with respect to addition and multiplication of functions defined point wise as below:

$$\left. \begin{aligned} (f+g)x &= f(x) + g(x) \\ \text{and } (fg)x &= f(x)g(x) \end{aligned} \right\} x \in [0, 1] \text{ where } f, g \in F$$
 (15 Marks)
28. Let a and b be elements of a group, with $a^2 = e, b^6 = e$ and $ab = b^4a$. Find the order of ab , and express its inverse in each of the forms $a^m b^n$ and $b^m a^n$ **(20 Marks)**

2010

29. Let $G = R - \{-1\}$ be the set of all real numbers omitting -1 . Define the binary relation $*$ on G by $a*b = a + b + ab$. Show $(G, *)$ is a group and it is abelian **(12 Marks)**
30. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify. **(12 Marks)**
31. Let (R^*, \cdot) be the multiplicative group of non-zero reals and $(GL(n, R), \cdot)$ be the multiplicative group of $n \times n$ non-singular real matrices. Show that the quotient group $\frac{GL(n, R)}{SL(n, R)}$ and (R^*, \cdot) are isomorphic where $SL(n, R) = \{A \in GL(n, R) / \det A = 1\}$ what is the center of $GL(n, R)$ **(15 Marks)**
32. Let $C = \{f : I = [0, 1] \rightarrow R / f \text{ is continuous}\}$. Show C is a commutative ring with 1 under point wise addition and multiplication. Determine whether C is an integral domain. Explain. **(15 Marks)**
33. Consider the polynomial ring $Q[x]$. Show $p(x) = x^3 - 2$ is irreducible over Q . Let I be the ideal $Q[x]$ in generated by $p(x)$. Then show that $\frac{Q[x]}{I}$ is a field and that each element of it is of the form $a_0 + a_1t + a_2t^2$ with a_0, a_1, a_2 in Q and $t = x + I$ **(15 Marks)**
34. Show that the quotient ring $\frac{Z[i]}{1+3i}$ is isomorphic to the ring $\frac{Z}{10Z}$ where $Z[i]$ denotes the ring of Gaussian integers **(15 Marks)**

2009

35. If R is the set of real numbers and R_+ is the set of positive real numbers, show that R under addition $(R, +)$ and R_+ under multiplication (R_+, \cdot) are isomorphic. Similarly if Q is set of rational numbers and Q_+ is the set of positive rational numbers, are $(Q, +)$ and (Q_+, \cdot) isomorphic? Justify your answer. **(4+8=12 Marks)**
36. Determine the number of homomorphisms from the additive group Z_{15} to the additive group Z_{10} (Z_n is the cyclic group of order n) **(12 Marks)**
37. How many proper, non-zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $Z_{12} \oplus Z_{12}$ have? Why? **(2+3+4+6=15Marks)**
38. Show that the alternating group of four letters A_4 has no subgroup of order 6. **(15 Marks)**
39. Show that $Z[X]$ is a unique factorization domain that is not a principal ideal domain (Z is the ring of integers). Is it possible to give an example of principal ideal domain that is not a unique factorization domain? ($Z[X]$ is the ring of polynomials in the variable X with integer.) **(15 Marks)**

40. How many elements does the quotient ring $\frac{\mathbb{Z}_5[X]}{X^2+1}$ have? Is it an integral domain? Justify your answers. (15 Marks)

2008

41. Let R_0 be the set of all real numbers except zero. Define a binary operation $*$ on R_0 as $a * b = |a|b$ where $|a|$ denotes absolute value of a . Does $(R_0, *)$ form a group? Examine. (12 Marks)
42. Suppose that there is a positive even integer n such that $a^n = a$ for all the elements a of some ring R . Show that $a + a = 0$ for all $a \in R$ and $a + b = 0 \Rightarrow a = b$ for all $a, b \in R$ (12 Marks)
43. Let G and \bar{G} be two groups and let $\phi: G \rightarrow \bar{G}$ be a homomorphism. For any element $a \in G$
- (i) Prove that $O(\phi(a))/O(a)$
 - (ii) $\text{Ker } \phi$ is normal subgroup of G . (15 Marks)
44. Let R be a ring with unity. If the product of any two non-zero elements is non-zero. Then prove that $ab = 1 \Rightarrow ba = 1$. Whether \mathbb{Z}_6 has the above property or not explain. Is \mathbb{Z}_6 an integral domain? (15 Marks)
45. Prove that every Integral Domain can be embedded in a field. (15 Marks)
46. Show that any maximal ideal in the commutative ring $F[x]$ of polynomial over a field F is the principal ideal generated by an irreducible polynomial. (15 Marks)

2007

47. If in a group G , $a^5 = e$, e is the identity element of G $aba^{-1} = b^2$ for $a, b \in G$, then find the order of b (12 Marks)
48. Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in \mathbb{Z}$. Show that R is a ring under matrix addition and multiplication $\left\{ A = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}, a, b \in \mathbb{Z} \right\}$. Then show that A is a left ideal of R but not a right ideal of R . (12 Marks)
49. (i) Prove that there exists no simple group of order 48. (15 Marks)
(ii) $1 + \sqrt{-3}$ and $\mathbb{Z}[\sqrt{-3}]$ is an irreducible element, but not prime. Justify your answer. (15 Marks)
50. Show that in the ring $R = \left\{ \frac{a + b\sqrt{-5}}{a}, b \in \mathbb{Z} \right\}$. The element $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R , where $\gamma = 7(1 + 2\sqrt{-5})$ (30 Marks)

2006

51. Let S be the set of all real numbers except -1. Define on S by $a * b = a + b + ab$. Is $(S, *)$ a group? Find the solution of the equation $2 * x * 3 = 7$ in S . (12 Marks)
52. If G is a group of real numbers under addition and N is the subgroup of G consisting of integers, prove that $\frac{G}{N}$ is isomorphic to the group H of all complex numbers of absolute value 1 under multiplication (12 Marks)
53. (i) Let $O(G) = 108$. Show that there exists a normal subgroup of order 27 or 9. (10 Marks)

(ii) Let G be the set of all those ordered pairs (a, b) of real numbers for which $a \neq 0$ and define in G , an operation as follows: $(a, b) \otimes (c, d) = (ac, bc + d)$ Examine whether G is a group w.r.t the operation \otimes . If it is a group, is G abelian? **(10 Marks)**

54. Show that $Z[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in Z\}$ is a Euclidean domain. **(30 Marks)**

2005

55. If M and N are normal subgroups of a group G such that $M \cap N = \{e\}$, show that every element of M commutes with every element of N . **(12 Marks)**

56. Show that $(1+i)$ is a prime element in the ring R of Gaussian integers. **(12 Marks)**

57. Let H and K be two subgroups of a finite group G such that $|H| > \sqrt{|G|}$ and $|K| > \sqrt{|G|}$. Prove that $H \cap K \neq \{e\}$. **(15 Marks)**

58. If $f: G \rightarrow G'$ is an isomorphism, prove that the order of $a \in G$ is equal to the order of $f(a)$. **(15 Marks)**

59. Prove that any polynomial ring $F[x]$ over a field F is U.F.D **(30 Marks)**

2004

60. If p is prime number of the form $4n+1$, n being a natural number, then show that congruence $x^2 \equiv -1 \pmod{p}$ is solvable. **(12 Marks)**

61. Let G be a group such that of all $a, b \in G$ (i) $ab = ba$ (ii) $(O(a), O(b)) = 1$ then show that $O(ab) = O(a) O(b)$ **(12 Marks)**

62. Verify that the set E of the four roots of $x^4 - 1 = 0$ forms a multiplicative group. Also prove that a transformation $T, T(n) = i^n$ is a homomorphism from I_+ (Group of all integers with addition) onto E under multiplication. **(10 Marks)**

63. Prove that if cancellation law holds for a ring R then $a(\neq 0) \in R$ is not a zero divisor and conversely **(10 Marks)**

64. The residue class ring $\frac{Z}{(m)}$ is a field iff m is a prime integer. **(15 Marks)**

65. Define irreducible element and prime element in an integral domain D with units. Prove that every prime element in D is irreducible and converse of this is not (in general) true. **(25 Marks)**

2003

66. If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G **(12 Marks)**

67. Show that the ring $Z[i] = \{a + ib/a, b \in Z, i = \sqrt{-1}\}$ of Gaussian integers is a Euclidean domain **(12 Marks)**

68. Let R be the ring of all real-valued continuous functions on the closed interval $[0, 1]$.
Let $M = \left\{ f(x) \in R / f\left(\frac{1}{3}\right) = 0 \right\}$. Show that M is a maximal ideal of R **(10 Marks)**

69. Let M and N be two ideals of a ring R . Show that $M \cup N$ is an ideal of R if and only if either $M \subseteq N$ or $N \subseteq M$ (10 Marks)
70. Show that $Q(\sqrt{3}, i)$ is a splitting field for $x^5 - 3x^3 + x^2 - 3$ where Q is the field of rational numbers (15 Marks)
71. Prove that $x^2 + x + 4$ is irreducible over F the field of integers modulo 11 and prove further that $\frac{F[x]}{(x^2 + x + 4)}$ is a field having 121 elements. (15 Marks)
72. Let R be a unique factorization domain (U.F.D), then prove that $R[x]$ is also U.F.D (10 Marks)

2002

73. Show that a group of order 35 is cyclic. (12 Marks)
74. Show that polynomial $25x^4 + 9x^3 + 3x + 3$ is irreducible over the field of rational numbers (12 Marks)
75. Show that a group of p^2 is abelian, where p is a prime number. (10 Marks)
76. Prove that a group of order 42 has a normal subgroup of order 7. (10 Marks)
77. Prove that in the ring $F[x]$ of polynomial over a field F , the ideal $1 = |p(x)|$ is maximal if and only if the polynomial $p(x)$ is irreducible over F . (20 Marks)
78. Show that every finite integral domain is a field (10 Marks)
79. Let F be a field with q elements. Let E be a finite extension of degree n over F . Show that E has q^n elements (10 Marks)

2001

80. Let K be a field and G be a finite subgroup of the multiplicative group of non-zero elements of K . Show that G is a cyclic group. (12 Marks)
81. Prove that the polynomial $1 + x + x^2 + x^3 + \dots + x^{p-1}$ where p is prime number is irreducible over the field of rational numbers. (12 Marks)
82. Let N be a normal subgroup of a group G . Show that $\frac{G}{N}$ is abelian if and only if for all $x, y \in G$, $xyz^{-1} \in N$ (20 Marks)
83. If R is a commutative ring with unit element and M is an ideal of R , then show that maximal ideal of R if and only if $\frac{R}{M}$ is a field (20 Marks)
84. Prove that every finite extension of a field is an algebraic extension. Give an example to show that the converse is not true. (20 Marks)

2000

85. Let n be a fixed positive integer and let Z_n be the ring of integers modulo n . Let $G = \{a \in Z_n \mid a \neq 0\}$ and a is relatively prime to n . Show that G is a group under multiplication defined in Z_n . Hence, or otherwise, show that $a^{\phi(n)} \equiv a \pmod{n}$ for all integers a relatively prime to n where $\phi(n)$ denotes the number of positive integers that are less than n and are relatively prime to n (20 Marks)

86. Let M be a subgroup and N a normal subgroup of group G . Show that MN is a subgroup of G and $\frac{MN}{N}$ is isomorphic to $\frac{M}{M \cap N}$. **(20 Marks)**
87. Let F be a finite field. Show that the characteristic of F must be a prime integer p and the number of elements in F must be p^m for some positive integer m . **(20 Marks)**
88. Let F be a field and $F[x]$ denote the set of all polynomials defined over F . If $f(x)$ is an irreducible polynomial in $F[x]$, show that the ideal generated by $f(x)$ in $F[x]$ is maximal and $\frac{F[x]}{f(x)}$ is a field. **(20 Marks)**
89. Show that any finite commutative ring with no zero divisors must be a field. **(20 Marks)**

1999

90. If ϕ is a homomorphism of G into \bar{G} with kernel K , then show that K is a normal subgroup of G . **(20 Marks)**
91. If p is prime number and $p^\alpha \mid O(G)$, then prove that G has a subgroup of order p^α . **(20 Marks)**
92. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field. **(20 Marks)**

1998

93. Prove that if a group has only four elements then it must be abelian. **(20 Marks)**
94. If H and K are subgroups of a group G then show that HK is a subgroup of G if and only if $HK = KH$. **(20 Marks)**
95. Let $(R, +, \cdot)$ be a system satisfying all the axioms for a ring with unity with the possible exception of $a + b = b + a$. Prove that $(R, +, \cdot)$ is a ring. **(20 Marks)**
96. If p is prime then prove that Z_p is a field. Discuss the case when p is not a prime number. **(20 Marks)**
97. Let D be a principal domain. Show that every element that its neither zero nor a unit in D is a product of irreducible. **(20 Marks)**

1997

98. Show that a necessary and sufficient condition for a subset H of a group G to be a subgroup is $HH^{-1} = H$. **(20 Marks)**
99. Show that the order of each subgroup of a finite group is a divisor of the order of the group. **(20 Marks)**
100. In a group G , the commutator (a, b) $a, b \in G$ is the element $aba^{-1}b^{-1}$ and the smallest subgroup containing all commutators is called the commutator subgroup of G . Show that a quotient group $\frac{G}{H}$ is abelian if and only if H contains the commutator subgroup of G . **(20 Marks)**
101. If $x^2 = x$ for all x in a ring R , show that R is commutative. Give an example to show that the converse is not true. **(20 Marks)**
102. Show that an ideal S of the ring of integers Z is maximal ideal if and only if S is generated by a prime integer. **(20 Marks)**
103. Show that in an integral domain every prime element is irreducible. Give an example to show that the converse is not true. **(20 Marks)**

1996

104. Let R be the set of real numbers and $G = \{(a, b) \mid a, b \in R, a \neq 0\}$. $G \times G \rightarrow G$ is defined by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a group. Is it abelian? **(20 Marks)**
105. Let f be a homomorphism of a group G onto a group G' with kernel H . For each subgroup K' of G' define K by. Prove that
- (i) K' is isomorphic to $\frac{K}{H}$
- (ii) $\frac{G}{K}$ is isomorphic to $\frac{G'}{K'}$ **(20 Marks)**
106. Prove that a normal subgroup H of a group G is maximal, if and only if the quotient group $\frac{G}{H}$ is simple. **(20 Marks)**
107. In a ring R , prove that cancellation laws hold. If and only if R has no zero divisors. **(20 Marks)**
108. If S is an ideal of ring R and T any subring of R , then prove that S is an ideal of $S + T = \{s + t \mid s \in S, t \in T\}$. **(20 Marks)**
109. Prove that the polynomial $x^2 + x + 4$ is irreducible over the field of integers modulo 11. **(20 Marks)**

1995

110. Let G be a finite set closed under an associative binary operation such that $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$. Prove that G is a group. **(20 Marks)**
111. Let G be group of order p^n , where p is a prime number and $n > 0$. Let H be a proper subgroup of G and $N(H) = \{x \in G : x^{-1}hx \in H \forall h \in H\}$. Prove that $N(H) \neq H$. **(20 Marks)**
112. Show that a group of order 112 is not simple. **(20 Marks)**
113. Let R be a ring with identity. Suppose there is an element a of R which has more than one right inverse. Prove that a has infinitely many right inverses. **(20 Marks)**
114. Let F be a field and let $p(x)$ be an irreducible polynomial over F . Let $\langle p(x) \rangle$ be the ideal generated by $p(x)$. Prove that $\langle p(x) \rangle$ is a maximal ideal. **(20 Marks)**
115. Let F be a field of characteristic $p \neq 0$. Let $F(x)$ be the polynomial ring. Suppose $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is an element of $F(x)$. Define $f(x) = a_1 + 2a_2x + 3a_3x^2 + \dots + na_nx^{n-1}$. If $f(x) = 0$, prove that there exists $g(x) \in F(x)$ such that $f(x) = g(x)^p$. **(20 Marks)**

1994

116. If G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers n for all $a, b \in G$, then prove that G is abelian. **(20 Marks)**
117. Can a group of order 42 be simple? Justify your claim **(20 Marks)**
118. Show that the additive group of integers modulo 4. Is isomorphic to the multiplicative group of the non-zero elements of integers modulo 5. State the two isomorphisms **(20 Marks)**
119. Find all the units of the integral domain of Gaussian integers. **(20 Marks)**
120. Prove or disprove the statement: The polynomial ring $I[x]$ over the ring of integers is a principal ideal ring. **(20 Marks)**

121. If R is an integral domain (not necessarily a unique factorization domain) and F is its field of quotients, then show that any element $f(x)$ in $F(x)$ is of the form $f(x) = \frac{f_0(x)}{a}$ where $f_0(x) \in R[x], a \in R$. (20 Marks)

1993

122. If G is a cyclic group of order n and p divides n , then prove that there is a homomorphism of G onto a cyclic group of order p . What is the Kernel of homomorphism? (20 Marks)
123. Show that a group of order 56 cannot be simple. (20 Marks)
124. Suppose that H, K are normal subgroups of a finite group G with H a normal subgroup of K . If $P = \frac{K}{H}, S = \frac{G}{H}$, then prove that the quotient groups $\frac{S}{P}$ and $\frac{G}{K}$ are isomorphic. (20 Marks)
125. If Z is the set of integers then show that $Z[\sqrt{-3}] = \{a + \sqrt{-3}b : a, b \in Z\}$ is not a unique factorization domain (20 Marks)
126. Construct the addition and multiplication table for $\frac{Z_3[x]}{\langle x^2 + 1 \rangle}$ where Z_3 is the set of integers modulo 3 and $\langle x^2 + 1 \rangle$ is the ideal generated by $(x^2 + 1)$ in $Z_3[x]$ (20 Marks)
127. Let Q be the set of rational number and $Q(2^{1/2}, 2^{1/3})$ the smallest extension field of Q containing $2^{1/2}, 2^{1/3}$. Find the basis for $Q(2^{1/2}, 2^{1/3})$ over Q . (20 Marks)

1992

128. If H is a cyclic normal subgroup of a group G , then show that every subgroup of H is normal in G . (20 Marks)
129. Show that no group of order 30 is simple. (20 Marks)
130. If p is the smallest prime factor of the order of a finite group G , prove that any subgroup of index p is normal. (20 Marks)
131. If R is unique factorization domain, then prove that any $f \in R[x]$ is an irreducible element of $R[x]$, if and only if either f is an irreducible element of R or f is an irreducible polynomial in $R[x]$. (20 Marks)
132. Prove that $x^2 + 1$ and $x^2 + x + 4$ are irreducible over F , the field of integers modulo 11. Prove also that $\frac{F[x]}{\langle x^2 + 1 \rangle}$ and $\frac{F[x]}{\langle x^2 + x + 4 \rangle}$ are isomorphic fields each having 121 elements. (20 Marks)
133. Find the degree of splitting field $x^5 - 3x^3 + x^2 - 3$ over Q , the field of rational numbers. (20 Marks)