

# Partial Differential Equations

Previous year Questions from  
2016 to 1992

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2017

# 2016

- Find the general equation of surfaces orthogonal to the family of spheres given by  $x^2 + y^2 + z^2 = cz$ . **(10 marks)**
- Find the general integral of the partial differential equation  $(y + zx)p - (x + yz)q = x^2 - y^2$  **(10 marks)**
- Determine the characteristics of the equation  $z = p^2 - q^2$  and find the integral surface which passes through the parabola  $4z + x^2 = 0$  **(15 marks)**
- Solve the partial differential equation  $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$  **(15 marks)**
- Find the temperature  $u(x, t)$  in a bar of silver of length and constant cross section of area  $1 \text{ cm}^2$ . Let density  $\rho = 10.6 \text{ g/cm}^3$ , thermal conductivity  $K = 1.04 \text{ (cm sec}^\circ \text{C)}$  and specific heat  $\sigma = 0.056 \text{ / g}^\circ \text{C}$  the bar is perfectly isolated laterally with ends kept at  $0^\circ \text{C}$  and initial temperature  $f(x) = \sin(0.1\pi x)^\circ \text{C}$  note that  $u(x, t)$  follows the heat equation  $u_t = c^2 u_{xx}$  where  $c^2 = k / (\rho\sigma)$  **(20 marks)**

# 2015

- Solve the partial differential equation:  $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$  where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$  **(10 Marks)**
- Solve:  $(D^2 + DD' - 2D'^2)u = e^{x+y}$ , where  $D = \frac{\partial}{\partial x}$  and  $D' = \frac{\partial}{\partial y}$  **(10 Marks)**
- Solve for the general solution  $p \cos(x + y) + q \sin(x + y) = z$ , where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$  **(15 Marks)**
- Find the solution of the initial-boundary value problem
 
$$u_t - u_{xx} + u = 0, \quad 0 < x < l, t > 0$$

$$u(0, t) = u(l, t) = 0, \quad t \geq 0$$

$$u(x, 0) = x(l - x), \quad 0 < x < l$$
**(15 Marks)**
- Reduce the second-order partial differential equation
 
$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$
 into canonical form. Hence, find its general solution **(15 Marks)**

# 2014

- Solve the partial differential equation  $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$  **(10 Marks)**
- Reduce the equation  $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$  to canonical form. **(15 Marks)**
- Find the deflection of a vibrating string (length =  $\pi$ , ends fixed,  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ ) corresponding to zero initial velocity and initial deflection.  $f(x) = k(\sin x - \sin 2x)$  **(15 Marks)**

14. Solve  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ ,  $0 < x < 1$ ,  $t > 0$ , given that

(i)  $u(x, 0) = 0$ ,  $0 \leq x \leq 1$ ;

(ii)  $\frac{\partial u}{\partial t}(x, 0) = x^2$ ,  $0 \leq x \leq 1$

(iii)  $u(0, t) = u(1, t) = 0$ , for all  $t$

(15 Marks)

## 2013

15. From a partial differential equation by eliminating the arbitrary functions  $f$  and  $g$  from  $z = yf(x) + xg(y)$

(10 Marks)

16. Reduce the equation  $y \frac{\partial^2 z}{\partial x^2} + (x + y) \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0$  to its canonical form when  $x \neq y$

(10 Marks)

17. Solve  $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$  where  $D$  and  $D'$  denote  $\frac{\partial}{\partial x}$  and  $\frac{\partial}{\partial y}$

(15 Marks)

18. Find the surface which intersects the surfaces of the system  $z(x + y) = C(3z + 1)$ , ( $C$  being a constant) orthogonally and which passes through the circle  $x^2 + y^2 = 1$ ,  $z = 1$

(15 Marks)

19. A tightly stretched string with fixed end points  $x = 0$  and  $x = l$  is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity  $\lambda x(l - x)$ , find the displacement of the string at any distance  $x$  from one end at any time  $t$

(20 Marks)

## 2012

20. Solve partial differential equation  $(D - 2D')(D - D')^2 z = e^{x+y}$

(12 Marks)

21. Solve partial differential equation  $px + qy = 3z$

(20 Marks)

22. A string of length  $l$  is fixed at its ends. The string from the mid-point is pulled up to a height  $k$  and then released from rest. Find the deflection  $y(x, t)$  of the vibrating string.

(20 Marks)

23. The edge  $r = a$  of a circular plate is kept at temperature  $f(\theta)$ . The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state.

(20 Marks)

## 2011

24. Solve the PDE  $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2 y$

(12 Marks)

25. Solve the PDE  $(x + 2z) \frac{\partial z}{\partial x} + (4zx - y) \frac{\partial z}{\partial y} = 2x^2 + y$

(12 Marks)

26. Find the surface satisfying  $\frac{\partial^2 z}{\partial x^2} = 6x + 2$  and touching  $z = x^3 + y^3$  along its section by the plane  $x + y + 1 = 0$ .

(20 Marks)

27. Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ,  $0 \leq x \leq a$ ,  $0 \leq y \leq b$  satisfying the boundary conditions

$$u(0, y) = 0, u(x, 0) = 0, u(x, b) = 0, \frac{\partial u}{\partial x}(a, y) = T \sin^3 \frac{\pi y}{a}$$

(20 Marks)

28. Obtain temperature distribution  $y(x, t)$  in a uniform bar of unit length whose one end is kept at  $10^0$  and the other end is insulated. Also it is given that  $y(x, 0) = 1 - x$ ,  $0 < x < 1$  **(20 Marks)**

## 2010

29. Solve the PDE  $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$  **(12 Marks)**
30. Find the surface satisfying the PDE  $(D^2 - 2DD' + D'^2)Z = 0$  and the conditions that  $bZ = y^2$  when  $x = 0$  and  $aZ = x^2$  when  $y = 0$  **(12 Marks)**
31. Solve the following partial differential equation

$$zp + yq = x$$

$$x_0(s) = s, y_0(s) = 1, z_0(s) = 2s$$

by the method of characteristics.

32. Reduce the following 2<sup>nd</sup> order partial differential equation into canonical form and find its general solution.  $xu_{xx} + 2x^2u_{xy} - u_x = 0$  **(20 Marks)**
33. Solve the following heat equation  
 $u_t - u_{xx} = 0$ ,  $0 < x < 2, t > 0$   
 $u(0, t) = u(2, t) = 0 \quad t > 0$   
 $u(x, 0) = x(2 - x), \quad 0 \leq x \leq 2$  **(20 Marks)**

## 2009

34. Show that the differential equation of all cones which have their vertex at the origin is  $px + qy = z$ . Verify that this equation is satisfied by the surface  $yz + zx + xy = 0$ . **(12 Marks)**
35. (i) Form the partial differential equation by elimination the arbitrary function  $f$  given by:  
 $f(x^2 + y^2, z - xy) = 0$  **(20 Marks)**
- (ii) Find the integral surface of:  $x^2p + y^2q + z^2r = 0$  which passes through the curve:  
 $xy = x + y, z = 1$  **(20 Marks)**
36. Find the characteristics of:  $y^2r - x^2t = 0$  where  $r$  and  $t$  have their usual meanings. **(15 Marks)**
37. Solve :  $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2) \sin xy - \cos xy$  where  $D$  and  $D'$  represent  $\frac{\partial}{\partial x}$  and  $\frac{\partial}{\partial y}$  **(15 Marks)**
38. A tightly stretched string has its ends fixed at  $x = 0$  and  $x = 1$ . At time  $t = 0$ , the string is given a shape defined by  $f(x) = \mu x(l - x)$ , where  $\mu$  is a constant, and then released. Find the displacement of any point  $x$  of the string at time  $t > 0$ . **(30 Marks)**

## 2008

39. Find the general solution of the partial differential equation  $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$  and also find the particular solution which passes through the lines  $x = 1, y = 0$  **(12 Marks)**

40. Find the general solution of the partial differential equation:  $(D^2 + DD' - 6D'^2)z = y \cos x$ , where  
 $D \equiv \frac{\partial}{\partial x}$ ,  $D' \equiv \frac{\partial}{\partial y}$  (12 Marks)
41. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines  $x = 0$ ,  $x = a$ ,  $y = 0$  and  $y = b$ . The edges and  $x = 0$ ,  $x = a$  and  $y = 0$  are kept at temperature zero while the edge  $y = b$  is kept at  $100^\circ\text{C}$ . (30 Marks)
42. Find complete and singular integrals of  $2xz - px^2 - 2qxy + pq = 0$  using Charpit's method. (15 Marks)
43. Reduce  $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$  canonical form. (15 Marks)

## 2007

44. (i) Form a partial differential equation by eliminating the function  $f$  from:  

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$
- (ii) Solve  $2zx - px^2 - 2qxy + pq = 0$  (6+6=12 Marks)
45. Transform the equation  $yz_x - xz_y = 0$  into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. (12 Marks)
46. Solve  $u_{xx} + u_{yy} = 0$  in  $D$  where  $D = \{(x, y) : 0 < x < a, 0 < y < b\}$  is a rectangle in a plane with the boundary conditions:  
 $u(x, 0) = 0$ ,  $u(x, b) = 0$ ,  $0 \leq x \leq a$   
 $u(0, y) = g(y)$ ,  $u_x(a, y) = h(y)$ ,  $0 \leq y \leq b$ . (30 Marks)
47. Solve the equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  by separation of variables method subject to the conditions:  
 $u(0, t) = 0 = u(l, t)$ , for all  $t$  and  $u(x, 0) = f(x)$  for all  $x$  in  $[0, l]$  (30 Marks)

## 2006

48. Solve :  $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$  (12 Marks)
49. Solve :  $\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = 2 \sin(3x + 2y)$  (12 Marks)
50. The deflection of vibrating string of length  $l$ , is governed by the partial differential equation  $u_{tt} = C^2 u_{xx}$ . The ends of the string are fixed at and  $x = 0$  and  $l$ . The initial velocity is zero. The initial displacement is given by  $u(x, 0) = \begin{cases} \frac{x}{l}, & 0 < x < \frac{l}{2} \\ \frac{1}{l}(l-x), & \frac{l}{2} < x < l. \end{cases}$   
 Find the deflection of the string at any instant of time. (30 Marks)
51. Find the surface passing through the parabolas  $z = 0$ ,  $y^2 = 4ax$  and  $z = 1$ ,  $y^2 = -4ax$  and satisfying the equation  $x \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial z}{\partial x} = 0$  (15 Marks)

52. Solve the equation  $p^2x + q^2y = z$ ,  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$  (15 Marks)

## 2005

53. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. (12 Marks)
54. Find the particular integral of  $x(y-z)p + y(z-x)q = z(x-y)$  which represents a surface passing through  $x = y = z$  (12 Marks)
55. The ends A and B of a rod 20cm long have the temperature at  $30^\circ\text{C}$  and  $80^\circ\text{C}$  until steady state prevails. The temperatures of ends are changed to  $40^\circ\text{C}$  and  $60^\circ\text{C}$  respectively. Find the temperature distribution in the rod at time  $t$ . (30 Marks)
56. Obtain the general solution of  $(D - 3D' - 2)^2z = 2e^{2x} \sin(y + 3x)$  where  $D = \frac{\partial}{\partial x}$  and  $D' = \frac{\partial}{\partial y}$  (30 Marks)

## 2004

57. Find the integral surface of the following partial differential equation :  
 $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$  (12 Marks)
58. Find the complete integral of the partial differential equation  $(p^2 + q^2)x = pz$  and deduce the solution which passes through the curve  $x = 0$ ,  $z^2 = 4y$ . (12 Marks)
59. Solve the partial differential equation :  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = (y-1)e^x$  (15 Marks)
60. A uniform string of length  $l$ , held tightly between  $x = 0$  and  $x = l$  with no initial displacement, is struck at  $x = a$ ,  $0 < a < l$ , with velocity  $v_0$ . Find the displacement of the string at any time  $t > 0$  (30 Marks)
61. Using Charpit's method, find the complete solution of the partial differential equation  
 $p^2x + q^2y = z$  (15 Marks)

## 2003

62. Find the general solution of  $\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y + \cos(2x + 3y)$  (12 Marks)
63. Show that the differential equations of all cones which have their vertex at the origin are  $px + qy = z$ . Verify that  $yz + zx + xy = 0$  is a surface satisfying the above equation. (12 Marks)
64. Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} = xy + e^{x+2y}$  (15 Marks)
65. Solve the equation  $p^2 - q^2 - 2px - 2qy + 2xy = 0$  using Charpit's method. Also find the singular solution of the equation, if it exists. (15 Marks)

66. Find the deflection  $u(x, t)$  of a vibrating string, stretched between fixed points  $(0, 0)$  and  $(3l, 0)$ , corresponding to zero initial velocity and following initial deflection:

$$f(x) = \begin{cases} \frac{hx}{l} & \text{when } 0 \leq x \leq l \\ \frac{h(3l-2x)}{l} & \text{when } l \leq x \leq 2l \\ \frac{h(x-3l)}{l} & \text{when } 2l \leq x \leq 3l \end{cases}$$

Where  $h$  is a constant.

(15 Marks)

## 2002

67. Find two complete integrals of the partial differential equation  $x^2p^2 + y^2q^2 - 4 = 0$  (12 Marks)
68. Find the solution of the equation  $z = \frac{1}{2}(p^2 + q^2) + (p-x)(q-y)$  (12 Marks)
69. Frame the partial differential equation by eliminating the arbitrary constants  $a$  and  $b$  from  $\log(az-1) = x + ay + b$  (10 Marks)
70. Find the characteristic strips of the equation  $xp + yq - pq = 0$  and then find the equation of the integral surface through the curve  $z = \frac{x}{2}, y = 0$  (20 Marks)
71. Solve:  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0$   
 $u(0, t) = u(l, t) = 0$   
 $u(x, 0) = x(l-x), 0 \leq x \leq l.$  (30 Marks)

## 2001

72. Find the complete integral partial differential equation  $2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2 + y^2)$  (12 Marks)
73. Find the general integral of the equation  $\{my(x+y) - nz^2\} \frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\} \frac{\partial z}{\partial y} = (lx - my)z$  (12 Marks)
74. Prove that for the equation  $z + px + qy - 1 - pqx^2y^2 = 0$  the characteristic strips are given by  $x(t) = \frac{1}{B + Ce^{-t}}, y(t) = \frac{1}{A + De^{-t}}, z(t) = E - (AC + BD)e^{-t}$   
 $p(t) = A(B + Ce^{-t})^2, q(t) = B(A + De^{-t})^2$  where  $A, B, C, D$  and  $E$  are arbitrary constants. Hence find the values of these arbitrary constants if the integral surface passes through the line  $z = 0, x = y$  (30 Marks)
75. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by  $x(x^2 + y^2 + z^2) = C_1y^2$  (10 Marks)
76. Solve the equation  $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = x^2y^4$  by reducing it to the equation with constant coefficients. (20 Marks)



# 2000

77. Solve :  $pq = x^m y^n z^{2l}$  (12 Marks)
78. Prove that if  $x_1^3 + x_2^3 + x_3^3 = 1$  when  $z = 0$ , the solution of the equation  $(S - x_1)p_1 + (S - x_2)p_2 + (S - x_3)p_3 = S - z$  can be given in the form  $S^3 \{(x_1 - z)^3 + (x_2 - z)^3 + (x_3 - z)^3\}^4 = (x_1 + x_2 + x_3 - 3z)^3$  where  $S = x_1 + x_2 + x_3 + z$  and  $p_i = \frac{\partial z}{\partial x_i}$ ,  $i = 1, 2, 3$ . (12 Marks)
79. Solve by Charpit's method the equation  $p^2 x(x-1) + 2pqxy + q^2 y(y-1) - 2pxz - 2qyz + z^2 = 0$  (15 Marks)
80. Solve :  $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{3x+4y}$ . (15 Marks)
81. A tightly stretched string with fixed end points  $x = 0$ ,  $x = l$  is initially at rest in equilibrium position. If it is set vibrating by giving each point  $x$  of it a velocity  $kx(l-x)$ , obtain at time  $t$  the displacement  $y$  at a distance  $x$  from the end  $x = 0$  (30 Marks)

# 1999

82. Verify that the differential equation  $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$  is integrable and find its primitive. (20 Marks)
83. Find the surface which intersects the surfaces of the system  $z(x+y) = c(3z+1)$ ,  $c$  is constant, orthogonally and which passes through the circle  $x^2 + y^2 = 1$ ,  $z = 1$  (20 Marks)
84. Find the characteristics of the equation  $pq = z$ , and determine the integral surface which passes through the parabola  $x = 0$ ,  $y^2 = z$ . (20 Marks)
85. Use Charpit's method to find a complete integral to  $p^2 + q^2 - 2px - 2qy + 1 = 0$  (20 Marks)
86. Find the solution of the equation  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-x} \cos y$  which  $\rightarrow 0$  as  $x \rightarrow \infty$  and has the value  $\cos y$  when  $x = 0$  (20 Marks)
87. One end of a string ( $x = 0$ ) is fixed, and the point  $x = a$  is made to oscillate, so that at time  $t$  the displacement is  $g(t)$ . Show that the displacement  $u(x, t)$  of the point  $x$  at time  $t$  is given by  $u(x, t) = f(ct-x) - f(ct+x)$  where  $f$  is a function satisfying the relation  $f(t+2a) = f(t) - g\left(\frac{t+a}{c}\right)$  (20 Marks)

# 1998

88. Find the differential equation of the set of all right circular cones whose axes coincide with the  $z$ -axis (20 Marks)
89. Form the differential equation by eliminating  $a, b$  and  $c$  from  $z = a(x+y) + b(x-y) + abt + c$  (20 Marks)
90. Solve  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = xyz$  (20 Marks)



91. Find the integral surface of the linear partial the differential equation  
 $x(y^2 + z) \frac{\partial z}{\partial x} - y(x^2 + z) \frac{\partial z}{\partial y} = (x^2 - y^2)z$  through the straight line  $x + y = 0, z = 1$  (20 Marks)
92. Use Charpit's method to find a complete integral of  $2x \left[ \left( z \frac{\partial z}{\partial y} \right)^2 + 1 \right] = z \frac{\partial z}{\partial x}$  (20 Marks)
93. Find a real function  $V(x, y)$ , which reduces to zero when  $y = 0$  and satisfies the equation  
 $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$  (20 Marks)
94. Apply Jacobi's method to find a complete integral of the equation  
 $2x \frac{\partial z}{\partial x_1} x_1 x_3 + 3 \frac{\partial z}{\partial x_2} x_3^2 + x \left( \frac{\partial z}{\partial x_2} \right)^2 x \frac{\partial z}{\partial x_3} = 0$  (20 Marks)

## 1997

95. (i) Find the differential equation of all surfaces of revolution having  $z$ -axis as the axis of rotation.  
 (ii) Form the differential equation by eliminating  $a$  and  $b$  from  $z = (x^2 + a)(y^2 + b)$  (10+10=20 Marks)
96. Find the equation of surfaces satisfying  $4yzp + q + 2y = 0$  and passing through  
 $y^2 + z^2 = 1, x + z = 2$  (20 Marks)
97. Solve :  $(y + z)p + (z + x)q = x + y$  (20 Marks)
98. Use Charpit's method to find complete integral of  $z^2(p^2 z^2 + q^2) = 1$  (20 Marks)
99. Solve :  $(D_x^3 - D_y^3)z = x^3 y^3$  (20 Marks)
100. Apply Jacobi's method to find complete integral of  $p_1^3 + p_2^2 + p_3 = 1$ . Here  
 $p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}, p_3 = \frac{\partial z}{\partial x_3}$  and  $z$  is a function of  $x_1, x_2, x_3$ . (20 Marks)

## 1996

101. (i) differential equation of all spheres of radius  $\lambda$  having their center in  $xy$ -plane  
 (ii) Form differential equation by eliminating  $f$  and  $g$  from  $z = f(x^2 - y) + g(x^2 + y)$  (10+10=20 Marks)
102. Solve :  $z^2(p^2 + q^2 + 1) = C^2$  (20 Marks)
103. Find the integral surface of the equation  $(x - y)y^2 p + (y - x)x^2 q = (x^2 + y^2)z$  passing through the curve  $xz = a^3, y = 0$  (20 Marks)
104. Apply Charpit's method to find the complete integral of  $z = px + ay + p^2 + q^2$  (20 Marks)
105. Solve :  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$  (20 Marks)
106. Find a surface passing through the lines  $z = x = 0$  and  $z - 1 = x - y = 0$  satisfying  
 $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 4 \frac{\partial^2 z}{\partial y^2} = 0$  (20 Marks)

# 1995

107. In the context of a partial differential equation of the first order in three independent variables, define and illustrate the terms:  
(i) The complete integral  
(ii) The singular integral (20 Marks)
108. Find the general integral of  $(y+z+w)\frac{\partial w}{\partial x} + (z+x+w)\frac{\partial w}{\partial y} + (x+y+w)\frac{\partial w}{\partial z} = x+y+z$  (20 Marks)
109. Obtain the differential equation of the surfaces which are the envelopes of a one-parameter family of planes. (20 Marks)
110. Explain in detail the Charpit's method of solving the nonlinear partial differential equation  
 $f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$  (20 Marks)
111. Solve  $\frac{\partial z}{\partial x_1} \frac{\partial z}{\partial x_2} \frac{\partial z}{\partial x_3} = z^3 x_1 x_2 x_3$  (20 Marks)
112. Solve  $(D_x^3 - 7D_x D_y^2 - 6D_y^3)z = \sin(x+2y) + e^{3x+y}$  (20 Marks)

# 1994

113. Find the differential equation of the family of all cones with vertex at  $(2, -3, 1)$  (20 Marks)
114. Find the integral surface of  $x^2 p + y^2 q + z^2 = 0$ ,  $p \equiv \frac{\partial z}{\partial x}$ ,  $q \equiv \frac{\partial z}{\partial y}$  which passes through the hyperbola  
 $xy = x + y, z = 1$  (20 Marks)
115. Obtain a Complete Solution of  $pq = x^m y^n z^{2l}$  (20 Marks)
116. Use the Charpit's method to solve  $16p^2 z^2 + 9q^2 z^2 + 4z^2 - 4 = 0$ . Interpret geometrically the complete solution and mention the singular solution. (20 Marks)
117. Solve  $(D^2 + 3DD' + 2D'^2)z = x + y$ , by expanding the particular integral in ascending powers of  $D$ , as well as in ascending powers of  $D'$ . (20 Marks)
118. Find a surface satisfying  $(D^2 + DD')z = 0$  and touching the elliptic paraboloid  $z = 4x^2 + y^2$  along its section by the plane  $y = 2x + 1$ . (20 Marks)

# 1993

119. Find the surface whose tangent planes cut off an intercept of constant length  $R$  from the axis of  $z$ . (20 Marks)
120. Solve  $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$  (20 Marks)
121. Find the integral surface of the partial differential equation  $(x-y)p + (y-x-z)q = z$  through the circle  $z = 1, x^2 + y^2 = 1$  (20 Marks)
122. Using Charpit's method find the complete integral of  $2xz - px^2 - 2qxy + pq = 0$  (20 Marks)
123. Solve  $r - s + 2q - z = x^2 y^2$  (20 Marks)
124. Find the general solution of  $x^2 r - y^2 t + xp - yq = \log x$  (20 Marks)

# 1992

125. Solve:  
 $(2x^2 - y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + 2z^2 - yz - zx - 2xy)$  (20 Marks)
126. Find the complete integral of  $(y - x)(qy - px) = (p - q)^2$  (20 Marks)
127. Use Charpit's method to solve  $px + qy = z\sqrt{1 + pq}$  (20 Marks)
128. Find the surface passing through the parabolas  $z = 0, y^2 = 4ax; z = 1, y^2 = -4ax$  and satisfying the differential equation  $xr + 2p = 0$  (20 Marks)
129. Solve :  $r + s - 6t = y \cos x$  (20 Marks)
130. Solve:  $\frac{\partial^2 u}{\partial x^2} \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y) + e^y$  (20 Marks)