RAMANASRI
IAS/IFoS Institute

Vector Analysis

Previous year Questions from 2016 to 1992

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2016

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2016

1. Prove that the vector $\vec{a} = 3\vec{i} + \hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}, \vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can from the sides of a triangle find the length of the medians of the triangle **(10 marks)**

2. Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$ **(10 marks)**

3. Prove that $\oint_C f d\vec{r} = \iint_S d\vec{S} \times \nabla f$ **(10 marks)**

4. For the of cardioid $r = a(1 + \cos\theta)$ show that the square of the radius of curvature at any point (r, θ) is proportion to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. **(15 marks)**

2015

5. Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ **(10 Marks)**

6. A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. **(12 Marks)**

7. Evaluate $\int_C e^{-x}(\sin y dx + \cos y dy)$, where C is the rectangle with vertices $(0,0)(\pi,0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ **(12 Marks)**

2014

8. Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude also. **(10 Marks)**

9. Evaluate by Stokes' theorem $\int_\Gamma (y dx + z dy + x dz)$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$ starting from $(2a, 0, 0)$ and then going below the z-plane. **(20 Marks)**

2013

10. Show the curve $\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. **(10 Marks)**

11. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator **(10 Marks)**

12. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$ **(10 Marks)**

13. By using Divergence Theorem of Gauss, evaluate the surface integral $\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-\frac{1}{2}} dS$, where S is the surface e of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. **(15 Marks)**

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14. Use Stokes' theorem to evaluate the line integral $\int_C (-y^3dx + x^3dy - z^3dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ **(15 Marks)**

2012

15. If $\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k},\ \vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$ find the value of $\frac{\partial^2}{\partial x \partial y}(\vec{A} + \vec{B})$ at $(1,0,-2)$ **(12 Marks)**

16. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Show that the curvature and torsion are equal for this curve. **(20 Marks)**

17. Verify Green's theorem in the plane for $\oint_C \left[xy + y^2dx + x^2dy\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ **(20 Marks)**

18. If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k},$, evaluate $\iint_S (\vec{\nabla} \times \vec{F}).\vec{n}d\vec{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy- plane. **(20 Marks)**

2011

19. For two vectors $\vec{a}$ and $\vec{b}$ give respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin 5t\hat{i} - \cos t\hat{j}$ determine:(i) $\frac{d}{dt}\left(\vec{a}.\vec{b}\right)$ and (ii) $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right)$ **(10 Marks)**

20. If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad v,$ find the value of $\vec{f} curl \vec{f}$ **(10 Marks)**

21. Examine whether the vectors $\nabla u, \nabla u$ and ∇w are coplanar, where u, v and w are the scalar functions defined by :

$$u = x + y + z,$$
$$v = x^2 + y^2 + z^2$$
$$\text{and } w = yz + zx + xy$$

(15 Marks)

22. If $\vec{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ calculate the double integral $\iint\left(\nabla \times \vec{u}\right)d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$ **(15 Marks)**

23. If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector. $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal **(15 Marks)**

24. Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 1$. **(15 Marks)**

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2010

25. Find the directional derivative of $f(x,y)=x^2y^3+xy$ at the point (2,1) in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis. **(12 Marks)**

26. Show that the vector field defined by the vector function $\vec{v}=xyz\left(yz\vec{i}+xy\vec{j}+xy\vec{k}\right)$ is conservative. **(12 Marks)**

27. Prove that $div\left(f\vec{V}\right)=f\left(div\vec{V}\right)+\left(grad.f\right)\vec{V}$ where f is a scalar function. **(20 Marks)**

28. Use the divergence theorem to evaluate $\iint_s \vec{V}\vec{n}dA$ where $\vec{V}=x^2z\vec{i}y\vec{j}-xz^2\vec{k}$ and S is he boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4y$. **(20 Marks)**

29. Verify Green's theorem for $e^{-x}\sin ydx+e^{-x}\cos y$ by the path of integration being the boundary of the square whose vertices are $(0,0),\left(\frac{\pi}{2},0\right)\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$ **(20 Marks)**

2009

30. Show that $div\left(gradr^n\right)=n(n+1)r^{n-2}$ where $r=\sqrt{x^2+y^2+z^2}$. **(12 Marks)**

31. Find the directional derivative of $(i)4xz^3-3x^2y^2z^2$ (i) at $(2,-1,2)$ along z-axis $(ii)-x^2yz+4xz^2$ at $(1,-2,1)$ in the direction of $2\hat{i}-\hat{j}-2\hat{k}$. **(6+6=12 Marks)**

32. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force of given by $\overline{F}=(2x-y+z)\hat{i}+\left(x+y-z^2\right)\hat{j}(3x-2y+4z)\hat{k}$. **(20 Marks)**

33. Using divergence theorem, evaluate $\iint_s \overline{A}.d\overline{S}$ where $\overline{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ **(20 Marks)**

34. Find the value of $\iint_s (\vec{\nabla}\times\vec{f}).d\vec{s}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$,when $\vec{F}=\left(y^2+z^2-x^2\right)\hat{i}+\left(z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-z^2\right)\hat{k}$ **(20Marks)**

2008

35. Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$. **(12 Marks)**

36. Show that $\vec{F}=\left(2xy+z^3\right)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from $(1-2,1)$ to $(3,1,4)$. **(12 Marks)**

37. Prove that $\nabla^2 f(x)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r)=0$. **(15 Marks)**

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38. Show that for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$ the curvature and torsion are same at every point. **(15 Marks)**

39. Evaluate $\int_c \vec{A} d\vec{r}$ along the curve $x^2 + y^2 = 1, z = 1$ form $(0,1,1)$ to $(1,0,1)$ if $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$. **(15 Marks)**

40. Evaluate $\iint_s \vec{F}\hat{n}d$ were $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}, \iint_s \vec{F}\hat{n}ds$ and S is the surface of the cylinder bounded by $x^2 + y^2 = 4z = 0$ and $z = 3$ **(15 Marks)**

2007

41. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$ determined grad(r_{-1}) in terms of $\hat{r}$ and r. **(12 Marks)**

42. Find the curvature and torsion at any point of the curve $x = a\cos 2t, y = a\sin 2t, z = 2a\sin t$. **(12 Marks)**

43. For any constant vector, show that the vector $\vec{a}$ represented by curl$(\vec{a} \times \vec{r})$ is always parallel to the vector $\vec{a}, \vec{r}$ being the position vector of a point (x, y, z) measured from the origin. **(15 Marks)**

44. If $\vec{r} = x\hat{i} + y\hat{j} + x\hat{k}$ find the value(s) of in order that $r^n\vec{r}$ may be (i) solenoidal (ii) irrotational **(15 Marks)**

45. Determine $\int_c (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2, x + y = 2a$ that starts from the point $(2a,0,0)$ goes at first below the z-plane. **(15 Marks)**

2006

46. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^2$ at the point $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to z-axis. **(12 Marks)**

47. If $\bar{A} = 2\bar{i} + \bar{K}, \bar{B} = \bar{i} + \bar{j} + \bar{K}, \bar{C} = 4\bar{i} - 3\bar{j} - 7\bar{K}$ determine a vector $\bar{R}$ satisfying the vector equation $\bar{R} \times \bar{B} = \bar{C} \times \bar{B} \,\&\, \bar{R}.\bar{A} = 0$ **(15 Marks)**

48. Prove that $r^n\bar{r}$ is an irrotational vector for any value o f n but is solenoidal only if $n + 3 = 0$ **(15Marks)**

49. If the unit tangent vector $\bar{t}$ and binormal $\bar{b}$ make angles ϕ and ϕ respectively with a constant unit vector $\bar{a}$ prove that $\frac{\sin\theta}{\sin\phi}.\frac{d\theta}{d\phi}. = -\frac{k}{\tau}$. **(15 Marks)**

50. Verify Stokes' theorem for the function $\bar{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$ and $y = a, a > 0$. **(15 Marks)**

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2005

51. Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\overrightarrow{AD}$ Hence find the volume of the tetrahedron with vertices $(2,2,2),(2,0,0),(0,2,0)$ and $(0,0,2)$ **(12 Marks)**
52. Prove that the curl of a vector field is independent of the choice of coordinates **(12 Marks)**
53. The parametric equation of a circular helix is $r = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$ where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured form $u = 0$ Also find $\frac{d\hat{t}}{ds}$ where s is the arc length. **(15 Marks)**
54. Show that $\operatorname{curl}\left(k\times\operatorname{grad}\frac{1}{r}\right)+\operatorname{grad}\left(k.\operatorname{grad}\frac{1}{r}\right)=0$ where r is the distance from the origin and k is the unit vector in the direction OZ **(15 Marks)**
55. Find the curvature and the torsion of the space curve **(15 Marks)**
56. Evaluate by Gauss divergence theorem, where S is the surface of the cylinder bounded by and **(15 Marks)**

2004

57. Show that if $\overline{A}$ and $\overline{B}$ are irrotational, then $\overline{A}\times\overline{B}$ is solenoidal. **(12 Marks)**
58. Show that the Frenet-Serret formulae can be written in the form $\frac{d\overline{T}}{ds}=\overline{\omega}\times\overline{T},\frac{d\overline{N}}{ds}=\overline{\omega}\times\overline{N}\ \&\ \frac{d\overline{B}}{dx}=\overline{\omega}\times\overline{B},$ where $\overline{\omega}=\tau\overline{T}+k\overline{B}$. **(12 Marks)**
59. Prove the identity $\nabla\left(\overline{A}.\overline{B}\right)=\left(\overline{B}.\nabla\right)\overline{A}+\left(\overline{A}.\nabla\right)\overline{B}+\overline{B}\times\left(\nabla\times\overline{A}\right)+\overline{A}\times\left(\nabla\times\overline{B}\right)$ **(15 Marks)**
60. Derive the identity $\iiint_v\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV=\iint_s\left(\phi\nabla\psi-\psi\nabla\phi\right)\hat{n}dS$ where V is the volume bounded by the closed surface S. **(15 Marks)**
61. Verify Stokes' theorem for $\hat{f}=(2x-y)\hat{i}-yz^2\hat{j}z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. **(15 Marks)**

2003

62. Show that if a' b' and c' are the reciprocals of the non-coplanar vectors a, b and c , then any vector r may be expressed as $r=(r.a')a+(r.b')b+(r.c')c.$ **(12 Marks)**
63. Prove that the divergence of a vector field is invariant w, r, to co-ordinate transformations. **(12 Marks)**
64. Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions. **(15 Marks)**
65. Prove the identity $\nabla A^2 = 2(A.\nabla)A+2A\times(\nabla\times A)$ where $\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$ **(15 Marks)**
66. Find the radii of curvature and torsion at a point of intersection of the surface $x^2-y^2=c^2, y=x\tanh\left(\frac{z}{c}\right).$ **(15 Marks)**

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67. Evaluate $\iint_s curtA.ds$ where S is the open surface $x^2+y^2-4x+4z=0, z\geq 0$ and $A=\left(y^2+z^2-x^2\right)\hat{i}+\left(2z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-3z^2\right)\hat{k}$. **(15 Marks)**

2002

68. Let $\overline{R}$ be the unit vector along the vector $\bar{r}(t)$ Show that $\overline{R}\times\frac{\overline{dR}}{dt}=\frac{\bar{r}}{r^2}\times\frac{\overline{dr}}{dt}$ where $r=\left|\bar{r}\right|$ **(12 Marks)**

69. Find the curvature k for the space curve $x=a\cos\theta, y=a\sin\theta, z=a\theta\tan\alpha$ **15 Marks)**

70. Show that $\left(cult\bar{v}\right)=grad\left(div\bar{v}\right)-\nabla^2\bar{v}$. **(15 Marks)**

71. Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it. Show that $\iint_s (fgradf).\hat{n}ds=\iiint_v\left[\left|gradf\right|^2+f\nabla^2 f\right]dv$ Hence $\iint_s (fgradf).\hat{n}ds$ or otherwise evaluate for $f=2x+y+2z$ over $s=x^2+y^2+z^2=4$ **(15 Marks)**

72. Find the values of constants a, b and c such that the maximum value of directional derivative of $f=axy^2+byz+cx^2z^2$ at $(1,-1,1)$ is in the direction parallel to y-axis and has magnitude 6. **(15 Marks)**

2001

73. Find the length of the arc of the twisted curve $r=\left(3t,3t^2,2t^3\right)$ from the point $t=0$ to the point $t=1$ Find also the unit tangent t, unit normal n and the unit binormal b at $t=1$. **(12 Marks)**

74. Show that $curl\frac{a\times r}{r^3}=-\frac{a}{r^3}+\frac{3r}{r^5}(a.r)$ where a is constant vector. **(12 Marks)**

75. Find the directional derivative of $f=x^2yz^3$ along $x=e^{-t}, y=1+2\sin t, z=t-\cos t$ at $t=0$ **(15 Marks)**

76. Show that the vector field defined by $F=2xyz^3i+x^2z^3j+3x^2yz^2k$ is irrotational. Find also the scalar u such that $F=grad\ u$ **(15 Marks)**

77. Verify Gauss' divergence theorem of $A=\left(4x,-2y^2,z^2\right)$ taken over the region bounded by $x^2+y^2=4, z=0$ and $z=3$. **(15 Marks)**

2000

78. In what direction from the point $(-1,1,1)$ is the directional derivative $f=x^2yz^3$ of a maximum? Compute its magnitude. **(12 Marks)**

79. Show that the covariant derivatives of the fundamental metric tensors $g_{ij}.g^{ij},\delta^i{}_j$ Vanish
(ii) Show that simultaneity is relative in special relativity theory. **(6+6=12 Marks)**

80. Show that
$(i)(A+B).(B+C)\times(C+A)=2A.B\times C$
$(ii)\nabla\times(A\times B)=(B.\nabla)A-B(\nabla.A)-(A.\nabla)B+A(\nabla.B)$ **(7+8=15 Marks)**

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81. Evaluate $\iint_S F.Nds$ where $F = 2xyi + yz^2 j + xzk$ and S is the surface of the parallelepiped bounded by $x = 0, y = 0, z = 0.x = 2.y = 1$ and $z = 3$ **(15 Marks)**

82. If g_{ij} and γ_{ij} are two metric tensors defined at a point and $\left\{ {l \atop ij} \right\}$ and Γ^{l}_{ij} are the corresponding Christoffel symbols of the second kind, then prove that $\left\{ {l \atop ij} \right\} - \Gamma^{l}_{ij}$ is a mixed tensor of the type A^{l}_{ij} **(15 Marks)**

83. Establish the formula $E = mc^2$ the symbols have their usual meaning. **(15 Marks)**

1999

84. If $\bar{a}, \bar{b}\bar{c}$ are the position vectors of A, B, C prove that $\bar{a} \times \bar{b} + \bar{b} + \bar{c} + \bar{c} \times \bar{a}$ is vector perpendicular to the plane ABC **(20 Marks)**

85. If $\bar{f} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right)$ find $\nabla \times \bar{F}$. **(20 Marks)**

86. Evaluate $\int_c \left(e^{-x} \sin ydx + e^{-x} \cos ydy\right)$ (by Green's theorem), where C is the rectangle whose vertices are $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ **(20 Marks)**

1998

87. If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point $P(x, y, z)$ then the values of grad $(r_1.r_2)$ and curl $(r_1 \times r_2)$. **(20 Marks)**

88. Show that $(a \times b) \times c = a \times (a \times b)$ if either $b = 0$ (or any other vector is 0) or c is collinear with a or b is orthogonal to a and c (both) **(20 Marks)**

89. Prove that $\left\{ {i \atop ik} \right\} = \frac{\partial}{\partial x_k}\left(\log \sqrt{g}\right)$. **(20 Marks)**

1997

90. Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are there given non-coplanar vectors $\vec{F}$ then any vector can be put in the form $F = \alpha \vec{B} \times \vec{C} + \beta \vec{C} \times \vec{A} + \gamma \vec{A} \times \vec{B}$ for given determine α, β, γ . **(20 Marks)**

91. Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ **(20 Marks)**

92. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij}T_{ij}$ of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti-symmetric part of T_{ij} . **(20 Marks)**

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1996

93. State and prove 'Quotient law' of tensors **(20 Marks)**

94. If $x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$ show that

$(i)\vec{r}\times grad f(r)=0$

$(ii) div(r^n\vec{r})=(n+3)r^n$ **(20 Marks)**

95. Verify Gauss's divergence theorem for $\vec{F}=xyx\hat{i}+z^2\hat{j}+2yz\hat{k}$ on the tetrahedron $x=y=z=,x+y+z=1$. **(20 Marks)**

1995

96. Consider a physical entity that is specified by twenty-seven numbers A_{ijk} in given coordinate system. In the transition to anther coordinates system of this kind. Let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk}-A_{ijk}$ are the components of a tensor B_{jk} of third order. Is the component of tensor? Give reasons for your answer **(20 Marks)**

97. Let the reason V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S. If ϕ is harmonic in V, show that $\int_s \frac{\partial\phi}{\partial n}ds=0$ **(20 Marks)**

98. In the vector field $u(x)$ let there exists a surface $curl v$ on which $v=0$. Show that, at an arbitrary point of this surface $curl v$ is tangential to the surface or vanishes. **(20 Marks)**

1994

99. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$.

(20 Marks)

100. If $\vec{F}=y\vec{i}+(x-2xz)\vec{j}-xy\vec{k}$ evaluate $\iint_s\left(\Delta\times\vec{F}\right)\vec{n}ds$ Where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy plane. **(20 Marks)**

101. Prove that $\left\{\begin{matrix} i \\ ik \end{matrix}\right\}=\frac{\partial}{\partial x}\left(log\sqrt{g}\right)$. **(20 Marks)**

1993

102. Prove that the angular velocity or rotation at any point is equal to one half of the curl of the velocity vector V. **(20 Marks)**

103. Evaluate $\iint_s \Delta\times\vec{F}\vec{n}ds$ where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k}$ **(20 Marks)**

104. Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor or rank one **(20 Marks)**

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1992

105. If $\vec{F}(x,y,z)=(y^2+z^2)\vec{i}+(z^2+x^2)\vec{j}+(x^2y^2)\vec{k}$ then calculate $\int_c \vec{f}\,\overrightarrow{dx}$ where C consist of

(i) The line segment form $(0,0,0)$ to $(1,1,1)$ (ii) the three line segments AB, BC and CD where A, B, C and D are respectively the points $(0,0,0),(1,0,0),(1,1,0)$ and $(1,1,1)$ (iii) the curve $\vec{x}+\overrightarrow{ui}+\overrightarrow{u^2j}+\overrightarrow{u^2k}, u$ from 0 to 1. **(20 Marks)**

106. If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

$(i) div\left\{x\times(\vec{a}\times\vec{x})\right\}=-2\vec{x}\vec{a}$

$(ii) div\left\{x\times(\vec{a}\times\vec{x})\times(\vec{b}\times\vec{x})\right\}=2\vec{a}(\vec{b}\times\vec{x})-2b(\vec{a}\times\vec{x})$ **(20 Marks)**

107. Obtain the formula $div\vec{A}=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{-1}}\left\{\left(\frac{g}{g_{ij}}\right)^{1/2}A(i)\right\}$ where $A(i)$ are physical components of $\vec{A}$

and use it to derive expression of $div\vec{A}$ in cylindrical polar coordinates **(20 Marks)**

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