RAMANASRI
IAS/IFoS Institute

Analytical Geometry

Previous year Questions from 2016 to 1992

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2017

1. Find the equation of the tangent at the point $(1,1,1)$ to the Conicoid $3x^2 - y^2 = 2z$. **[10 Marks]**
2. Find the shortest distance between the skew the lines:

   $\frac{x-3}{3} = \frac{8-y}{1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ **[10 Marks]**

   A plane through a fixed point (a,b,c) and cuts the axes at the points A,B,C respectively. Find the locus of the center of the sphere which passes through the origin O and A,B,C **[15 Marks]**

   Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ find the point of contact. **[10 Marks]**

   Find the locus of the points of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1.$ **[10 Marks]**

   Reduce the following equation to the standard form and hence determine the nature of the Conicoid: $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. **[15 Marks]**

3. A plane through a fixed point (a,b,c) and cuts the axes at the points A,B,C respectively. Find the locus of the center of the sphere which passes through the origin O and A,B,C **[15 Marks]**
4. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ find the point of contact. **[10 Marks]**
5. Find the locus of the points of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1.$ **[10 Marks]**
6. Reduce the following equation to the standard form and hence determine the nature of the Conicoid: $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. **[15 Marks]**

2016

7. Find the equation of the sphere which passes though the circle $x^2 + y^2 = 4; z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. **[10 marks]**
8. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$ for what value of will the two lines intersect? **[10 marks]**
9. Find the surface generated by a line which intersects the line $y = a = z, x + 3z = a = y + z$ and parallel to the plane $x + y = 0$. **[10 marks]**
10. Show that the cone $3yz - 2zx - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{z}$ is a generator belonging to one such set, Find the other two. **[10 marks]**
11. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the Conicoid $ax^2 + by^2 + cz^2 = 1$. **[15 marks]**

2015

12. Find what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. **[10 Marks]**
13. If $6x = 3y = 2z$ represents one of the mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators. **[13 Marks]**
14. Obtain the equation of the plane passing through the points $(2,3,1)$ and $(4,-5,3)$ parallel to x- axis **[6 Marks]**
15. Verify if the lines: $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar. If yes, find the equation of the plane in which they lie. **[7 Marks]**
16. Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches. **[13 Marks]**

2014

17. Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines **[10 Marks]**
18. Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4,$ the tangent planes at which are parallel to the plane $2x - y + 2z = 1$ **[10 Marks]**
19. Prove that equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ **[10 Marks]**
20. Show that the lines drawn from the origin parallel to the normals to the central Conicoid $ax^2 + by^2 + cz^2 = 1,$ at its points of intersection with the plane $lx + my + nz = p$ generate the cone
$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2$$
[15 Marks]
21. Find the equations of the two generating lines through any point $(a\cos\theta, b\sin\theta, 0)$ of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ of the hyperboloid by the plane $z = 0$ **[15 Marks]**

2013

22. Find the equation of the plane which passes through the points $(0,\ 1,\ 1)$ and $(2,\ 0,\ -1)$ and is parallel to the line joining the points $(-1,\ 1,\ -2), (3,\ -2,\ 4)$. Find also the distance between the line and the plane. **[10 Marks]**
23. A sphere S has points $(0,\ 1,\ 0)$ $(3,\ -5,\ 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. **[10 Marks]**
24. Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$ **[15 Marks]**
25. A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0,\ z = 0$ and passes through a fixed point $(0,\ 0,\ \mathrm{c})$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that vertex lies one the fixed circle $x^2 + y^2 + 2ax + 2by = 0, 2ax + 2by + cz = 0$ **[15 Marks]**
26. A variable generator meets two generators of the system through the extremities B and B^1 of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2c^2 = 1$ in P and P^1 Prove that $BP.\ P^1B^1 = a^2 + c^2$ **[20 Marks]**

2012

27. Prove that two of the straight lines represented by the equation $x^3 + bx^2y + cxy^2 + y^3 = 0$ will be at right angles, if $b + c = -2$ **[12 Marks]**

28. A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C respectively. Prove that circle ABC lies on the cone $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$ **[20 Marks]**

29. Show that locus of a point from which three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z^2 = 0$ is $x^2 + y^2 + 4z = 1$ **[20 Marks]**

2011

30. Find the equation of the straight line through the point $(3,\ 1,\ 2)$ to intersect the straight line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$ **[10 Marks]**

31. Show that the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point $(1,\ 2,\ -2)$ and the passes through the point $(-1,\ 0,\ 0)$ is $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ **[10 Marks]**

32. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3,\ 1,\ -1)$ **[20 Marks]**

33. Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that lines joining to P, Q and R to origin are mutually perpendicular. Prove that plane PQR touches a fixed sphere **[20 Marks]**

34. Show that the cone $yz + xz + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area **[20 Marks]**

35. Show that generators through any one of the ends of an equi-conjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are inclined to each other at an angle of 60^0 if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. **[20 Marks]**

2010

36. Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle **[12 Marks]**

37. Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact **[20 Marks]**

38. Show that every sphere through the circle $x^2 + y^2 - 2ax + r^2 = 0,\ z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2, y = 0$ **[20 Marks]**

39. Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid $\frac{x^2}{4} + y^2 - z^2 = 49$ passing through $(10,\ 5,\ 1)$ and $(14,\ 2,\ -2)$ **[20 Marks]**

2009

40. A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0$ to meet two fixed lines $y=mx,\ z=c$ and $y=-mx,\ z=-c$. Find the locus of the line **[12 Marks]**
41. Find the equation of the sphere having its center on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-12x-3y+4z+8=0,\ 3x+4y-5z+3=0$ **[12 Marks]**
42. Prove that the normals from the point $(\alpha,\ \beta,\ \gamma)$ to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$ **[20 Marks]**

2008

43. The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$; find the equation of the plane in its new position **[12 Marks]**
44. Find the equations (in symmetric form) of the tangent line to the sphere $x^2+y^2+z^2+5x-7y+2z-8=0$, $3x-2y+4z+3=0$ at the point $(-3,\ 5,\ 4)$. **[12 Marks]**
45. A sphere S has points $(0,\ 1,\ 0),(3,\ -5,\ 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle **[20 Marks]**
46. Show that the enveloping cylinders of the ellipsoid $a^2x^2+b^2y^2+c^2z^2=1$ with generators perpendicular to z-axis meet the plane $z=0$ in parabolas **[20 Marks]**

2007

47. Find the equation of the sphere inscribed in the tetrahedron whose faces are $x=0,\ y=0,\ z=0$ and $2x+3y+6z=6$ **[12 Marks]**
48. Find the focus of the point which moves so that its distance from the plane $x+y-z=1$ is twice its distance from the line $x=-y=z$ **[12 Marks]**
49. Show that the spheres $x^2+y^2+z^2-x+z-2=0$ and $3x^2+3y^2-8x-10y+8z+14=0$ cut orthogonally. Find the center and radius of their common circle **[15 Marks]**
50. A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{4}$ and $\frac{x-11}{3}=\frac{y-5}{-1}=\frac{z}{1}$. Find the coordinate of the points of intersection and the length intercepted on it **[15 Marks]**
51. Show that the plane $2x-y+2z=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines **[15 Marks]**
52. Show that the feet of the normals from the point $P(\alpha,\ \beta,\ \gamma),\beta\neq 0$ on the paraboloid $x^2+y^2=4z$ lie on the sphere $2\beta(x^2+y^2+z^2)-(\alpha^2+\beta^2)y-2\beta(2+\gamma)z=0$ **[15 Marks]**

2006

53. A pair of tangents to the conic $ax^2+by^2=1$ intercepts a constant distance $2k$ on the y- axis. Prove that the locus of their point of intersection is the conic $ax^2(ax^2+by^2-1)=bk^2(ax^2-1)^2$ **[12 Marks]**
54. Show that the length of the shortest distance between the line $z=x\tan\alpha, y=0$ and any tangent to the ellipse $x^2\sin^2\alpha+y^2=a^2, z=0$ is constant **[12 Marks]**

55. If PSP^1 and QSQ^1 are the two perpendicular focal chords of a conic $\frac{1}{r}=1+e\cos\theta$, Prove that $\frac{1}{SP.SP^1}+\frac{1}{SQ.SQ^1}$ is constant **[15 Marks]**

56. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point $(1,\ -2,\ 1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$ **[15 Marks]**

57. Show that the plane $ax+by+cz=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines, if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ **[15 Marks]**

58. If the plane $lx+my+nz=p$ passes through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ prove that $a^2l^2+b^2m^2+c^2n^2=3p^2$ **[15 Marks]**

2005

59. If normals at the points of an ellipse whose eccentric angles are α,β,γ and δ in a point then show that $\sin(\beta+\gamma)+\sin(\gamma+\alpha)+\sin(\alpha+\beta)=0$ **[12 Marks]**
60. A square $ABCD$ having each diagonal AC and BD of length $2a$ is folded along the diagonal AC so that the planes DAC and BAC are at right angle. Find the shortest distance between AB and DC **[12 Marks]**
61. A plane is drawn through the line $x+y=1, z=0$ to make an angle $\sin^{-1}\left(\frac{1}{3}\right)$ with plane $x+y+z=5$. Show that two such planes can be drawn. Find their equations and the angle between them. **[15 Marks]**
62. Show that the locus of the centers of sphere of a co-axial system is a straight line. **[15 Marks]**
63. Obtain the equation of a right circular cylinder on the circle through the points $(a,\ 0,\ 0),(0,\ \mathrm{b},\ 0),(0,\ 0,\ \mathrm{c})$ as the guiding curve. **[15 Marks]**
64. Reduce the following equation to canonical form and determine which surface is represented by it: $x^2-7y^2+2z^2-10yz-8zx-10xy+6x+12y-6z+2=0$ **[15 Marks]**

2004

65. Prove that the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2=4ax$ is $(x+a)y^2+x^3=0$ **[12 Marks]**
66. Find the equations of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0,$ which are parallel to the plane $2x+y-z=4$ **[12 Marks]**
67. Find the locus of the middle points of the chords of the rectangular hyperbola $x^2-y^2=a^2$ which touch the parabola $y^2=4ax$ **[15 Marks]**
68. Prove that the locus of a line which meets the lines $y=\pm mx,\ z=\pm c$ and the circle $x^2+y^2=a^2,\ z=0$ is $c^2m^2(cy-mzx)^2+c^2(yz-cmx)^2=a^2m^2(z-c^2)^2$ **[15 Marks]**
69. Prove that the lines of intersection of pairs of tangent planes to $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone $a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0$ **[15 Marks]**
70. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point $(\alpha,\ \beta,\ \gamma)$. Prove that the perpendiculars to them through the origin generate the cone $(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$ **[15 Marks]**

2003

71. A variable plane remains at a constant distance unity from the point $(1,\ 0,\ 0)$ and cuts the coordinate axes at $A, B,$ and $C,$ find the locus of the center of the sphere passing through the origin and the point and the point A, B and C. **[12 Marks]**
72. Find the equation of the two straight lines through the point $(1,\ 1,\ 1)$ that intersect the line $x-4=4(y-4)=2(z-1)$ at an angle of 60^0 **[12 Marks]**
73. Find the volume of the tetrahedron formed by the four planes $lx+my+nz=p, lx+my=0, my+nz=0$ and $nz+lx=0$ **[15 Marks]**
74. A sphere of constant radius r passes through the origin O and cuts the co-ordinate axes at A, B and C. Find the locus of the foot of the perpendicular from O to the plane ABC. **[15 Marks]**
75. Find the equations of the lines of intersection of the plane $x+7y-5z=0$ and the cone $3xy+14zx-30xy=0$ **[15 Marks]**
76. Find the equations of the lines of shortest distance between the lines: $y+z=1,\ x=0$ and $x+z=1,\ y=0$ as the intersection of two planes **[15 Marks]**

2002

77. Show that the equation $9x^2-16y^2-18x-32y-151=0$ represents a hyperbola. Obtain its eccentricity and foci. **[12 Marks]**
78. Find the co-ordinates of the center of the sphere inscribed in the tetrahedron formed by the plane $x=0,\ y=0,\ z=0$ and $x+y+z=a$ **[12 Marks]**
79. Tangents are drawn from any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the circle $x^2+y^2=r^2$. Show that the chords of contact are tangents to the ellipse $a^2x^2+b^2y^2=r^2$ **[15 Marks]**
80. Consider a rectangular parallelepiped with edges a, b and c. Obtain the shortest distance between one of its diagonals and an edge which does not intersect this diagonal **[15 Marks]**
81. Show that the feed of the six normals drawn from any point $(\alpha,\ \beta,\ \gamma)$ to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the cone $\frac{a^2(b^2-c^2)\alpha}{x}+\frac{b^2(c^2-a^2)\beta}{y}+\frac{c^2(a^2-b^2)\gamma}{z}=0$ **[15 Marks]**
82. A variable plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ is parallel to the plane meets the co-ordinate axes of A, B and C. Show that the circle ABC lies on the conic $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$ **[15 Marks]**

2001

83. Show that the equation $x^2-5xy+y^2+8x-20y+15=0$ represents a hyperbola. Find the coordinates of its center and the length of its real semi-axes. **[12 Marks]**
84. Find the shortest distance between the axis of z and the lines $ax+by+cz+d=0,\ a^1x+b^1y+c^1z+d^1=0$ **[12 Marks]**
85. Find the equation of the circle circumscribing the triangle formed by the points $(a,\ 0,\ 0), (0,\ \mathrm{b},\ 0), (0,\ 0,\ \mathrm{c})$. Obtain also the coordinates of the center of the circle. **[15 Marks]**
86. Find the locus of equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **[15 Marks]**

87. Prove that $5x^2+5y^2+8z^2+8yz+8zx-2xy+12x-12y+6=0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$ **[15 Marks]**

88. If TP, TQ and T^1P^1, T^1Q^1 all lie one a conic. **[15 Marks]**

2000

89. Find the equations to the planes bisecting the angles between the planes $2x-y-2z=0$ and $3x+4y+1=0$ and specify the on e which bisects the acute angle. **[12 Marks]**

90. Find the equation to the common conjugate diameters of the conics $x^2+4xy+6y^2=1$ and $2x^2+6xy+9y^2=1$ **[12 Marks]**

91. Reduce the equation $x^2+y^2+z^2-2xy-2yz+2zx+x-y-2z+6=0$ into canonical form and determine the nature of the quadric **[15 Marks]**

92. Find the equation of the sphere through the circle $x^2+y^2+z^2=4, x+2y-z=2$ and the point $(1,-1,1)$ **[15 Marks]**

93. A variable straight line always intersects the lines $x=c, y=0; y=c, z=0; z=c, x=0$. Find the equations to its locus **[15 Marks]**

94. Show that the locus of mid-points of chords of the cone $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ drawn parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is the plane $(al+hm+gn)x+(hl+bm+fn)y+(gl+fm+cn)z=0$ **[20 Marks]**

1999

95. If P and D are ends of a pair of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ show that the tangents at P and D meet on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ **[20 Marks]**

96. Find the equation of the cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are perpendicular to the plane $x-y-3z=5$ **[20 Marks]**

1998

97. Find the locus of the pole of a chord of the conic $\frac{l}{r}=1+e\cos\theta$ which subtends a constant angle 2α at the focus **[20 Marks]**

98. Show that the plane $ax+by+cz+d=0$ divides the join of $P_1\equiv(x_1, y_1, z_1), P_2\equiv(x_2, y_2, z_2)$ in the ratio $-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$. Hence show that the planes $U\equiv ax+by+cz+d=0=a^1x+b^1y+c^1z+d^1\equiv V$, $U+\lambda V=0$ and $U-\lambda V=0$ divide any transversal harmonically **[20 Marks]**

99. Find the smallest sphere (i.e. the sphere of smallest radius) which touches the lines $\frac{x-5}{2}=\frac{y-2}{-1}=\frac{z-5}{-1}$ and $\frac{x+4}{-3}=\frac{y+5}{-6}=\frac{z-4}{4}$ **[20 Marks]**

100. Find the co-ordinates the point of intersection of the generators $\frac{x}{a}-\frac{y}{b}-2\lambda=0=\frac{x}{a}-\frac{y}{b}-\frac{z}{\lambda}$ and $\frac{x}{a}+\frac{y}{b}-2\mu=0=\frac{x}{a}-\frac{y}{b}-\frac{z}{\mu}$ of the surface $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z.$ Hence show that the locus of the points of intersection of perpendicular generators curves of intersection of the surface with the plane $2z+(a^2-b^2)=0$ **[20 Marks]**

101. Let $P\equiv(x',\ y',\ z')$ lie on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$ If the length of the normal chord through P is equal to $4PG$, where G is the intersection of the normal with the z- plane, then show that P lies on the cone $\frac{x^2}{a^6}(ac^2-a^2)+\frac{y^2}{b^6}(ac^2-b^2)+\frac{z^2}{c^4}=0$ **[20 Marks]**

1997

102. Let P be a pint on an ellipse with its center at the point C. Let CD and CP be two conjugate diameters. If the normal at P cuts CD in F, show that $CD.PF$ is a constant and the locus of F is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=\left[\frac{a^2-b^2}{x^2+y^2}\right]^2$ where $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ equation of the given ellipse **[20 Marks]**

103. A circle passing through the focus of conic section whose latus rectum is $2l$ meets the conic in four points whose distances from the focus are $\gamma_1,\gamma_2,\gamma_3$ and γ_4 Prove that $\frac{1}{\gamma_1}+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}+\frac{1}{\gamma_4}=\frac{2}{l}$ **[20 Marks]**

104. Find the reflection of the plane $x+y+z-1=0$ in plane $3x+4z+1=0$ **[20 Marks]**

105. Show that the pint of intersection of three mutually perpendicular tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lies on the sphere $x^2+y^2+z^2=a^2+b^2+c^2$ **[20 Marks]**

106. Find the equation of the spheres which pass through the circle $x^2+y^2+z^2-4x-y+3z+12=0,$ $2x+3y-7z=10$ and touch the plane $x-2y+2z=1$ **[20 Marks]**

1996

107. A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Through A,B,C the planes are drawn parallel to the coordinate planes. Show that the locus of their point of intersection is given by $x^{-2}+y^{-2}+z^{-2}=p^{-2}$ **[20 Marks]**

108. Find the equation of the sphere which passes through the points $(1,0,0),(0,1,0),(0,0,1)$ and has the smallest possible radius. **[20 Marks]**

109. The generators through a point P on the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meet the principal elliptic section in two points such that the eccentric angle of one is double that of the other. Show that P lies on the curve $x=\frac{a(1-3t^2)}{1+t^2},y=\frac{bt(3-t^2)}{1+t^2},z=ct$ **[20 Marks]**

1995

110. Two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the circle $x^2+y^2=r^2$ at P and Q. Show that the locus of middle point of PQ is $a^2\left\{(x^2+y^2)^2-r^2x^2\right\}+b^2\left\{(x^2+y^2)^2-r^2y^2\right\}=0$ **[20 Marks]**

111. If the normal at one of the extremities of latus rectum of the conic $\frac{1}{r}=1+e\cos\theta$, meets the curve again at Q, show that $SQ=\frac{l(1+3e^2+e^4)}{(1+e^2-e^4)}$, where S is the focus of the conic. **[20 Marks]**

112. Through a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes in A,B,C. Prove that the area of the triangle ABC is $\frac{r^2}{2x'y'z'}$ where r is the measure of OP. **[20 Marks]**

113. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the area of the common circle is $\frac{\pi r_1^2 r_2^2}{r_1^2+r_2^2}$ **[20 Marks]**

114. Show that a plane through one member of the λ- system and one member of μ- system is tangent plane to the hyperboloid at the point of intersection of the two generators. **[20 Marks]**

1994

115. If 2φ be the angle between the tangents from $P(x_1,y_1)$ to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, prove that $\lambda_1\cos^2\varphi+\lambda_2\sin^2\varphi=0$ where λ_1,λ_2 are the parameters of two confocals to the ellipse through P **[20 Marks]**

116. If the normals at the points $\alpha,\beta,\gamma,\delta$ on the conic $\frac{1}{r}=1+e\cos\theta$ meet at (ρ,ϕ), prove that $\alpha+\beta+\gamma+\delta-2\phi=$ odd multiple of π radians. **[20 Marks]**

117. A variable plane is at a constant distance p from the origin O and meets the axes in A,B and C. Show that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$ **[20 Marks]**

118. Find the equations to the generators of hyperboloid, through any point of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, z=0$ **[20 Marks]**

119. Planes are drawn through a fixed point (α,β,γ) so that their sections of the paraboloid $ax^2+by^2=2z$ are rectangular hyperbolas. Prove that they touch the cone $\frac{(x-\alpha^2)}{b}+\frac{(y-\beta^2)}{a}+\frac{(z-\gamma^2)}{a+b}=0$. **[20 Marks]**

1993

120. Two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cut the circle $x^2+y^2=r^2$ at P and Q. Show that the locus of the middle points of PQ is $a^2\left\{(x^2+y^2)^2-r^2x^2\right\}+b^2\left\{(x^2+y^2)^2-r^2y^2\right\}=0$ **[20 Marks]**

121. If the normal at one of the extremities of latus rectum of the conic $\frac{1}{r}=1+e\cos\theta$, meets the curve again at Q, show that $SQ=\frac{l(1+3e^2+e^4)}{(1+e^2-e^4)}$ where S is the focus of the conic. **[20 Marks]**

122. Through a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes in A,B,C. Prove that the area of the triangle ABC is $\frac{r^2}{2x'y'z'}$, where r is the measure of OP **[20 Marks]**

123. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the area of the common circle is $\frac{\pi r_1^2 r_2^2}{r_1^2+r_2^2}$ **[20 Marks]**

124. Show that a plane through one member of λ- system and one member μ- system is tangent plane to the hyperboloid at the pint of intersection of the two generators **[20 Marks]**

1992

125. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two intersecting lines, show that the square of the distance of the point of intersection of the straight lines from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}(ab-h^2\neq 0)$ **[20 Marks]**

126. Discuss the nature of the conic $16x^2-24xy+9y^2-104x-172y+144=0$ in detail **[20 Marks]**

127. A straight line, always parallel to the plane of yz, passed through the curves $x^2+y^2=a^2,\ z=0$ and $x^4=ax,\ y=0$ prove that the equation of the surface generated is $x^4y^2=(x^2-az)^2(a^2-x^2)$ **[20 Marks]**

128. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point $(\alpha,\ \beta,\ \gamma)$. Prove that the perpendicular them from the origin generate the cone $(\alpha x,\ \beta y,\ \gamma z)^2=a^2x^2+b^2y^2+c^2z^2$ **[20 Marks]**

129. Show that the locus of the foot of the perpendicular from the center to the plane through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $a^2x^2+b^2y^2+c^2z^2=3(x^2+y^2+z^2)$ **[20 Marks]**