

Modern Algebra

Previous year Questions
from 2017 to 1992

Ramanasri

2017

2017

1. Let G be a group of order n . Show that G is isomorphic to a subgroup of the permutation group S_n . [10 Marks]
2. Let F be a field and $F[x]$ denote the ring of polynomial over F in a single variable X . For $f(X), g(X) \in F[X]$ with $g(X) \neq 0$, show that there exist $q(X), r(X) \in F[X]$ such that $\text{degree } r(X) < \text{degree } g(X)$ and $f(X) = q(X).g(X) + r(X)$. [20 Marks]
3. Show that the groups $Z_5 \times Z_7$ and Z_{35} are isomorphic. [15 Marks]

2016

1. Let K be a field and $K[X]$ be the ring of polynomials over K in a single variable X for a polynomial $f \in K[X]$ Let (f) denote the ideal in $K[X]$ generated by f . show that (f) is a maximal ideal in $K[X]$ if and only iff is an irreducible polynomial over K . (10 marks)
2. Let p be prime number and Z_p denote the additive group of integers modulo p . show that that every non-zero element Z_p of generates Z_p (15 marks)
3. Let K be an extension of a field F prove that the element of K which are algebraic over F form a subfield of K Further if $F \subset K \subset L$ are fields L is algebraic over K and K is algebraic over F then prove that L is algebraic over F . (20 marks)
4. Show that every algebraically closed field is infinite. (15 marks)

2015

5. (i) How many generators are there of the cyclic group G of order 8? Explain. (5 Marks)
(ii) Taking a group $\{e, a, b, c\}$ of order 4, where e is the identity, construct composition tables showing that one is cyclic while the other is not (5 Marks)
6. Give an example of a ring having identity but a subring of this having a different identity. (10 Marks)
7. If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R' , prove that $\phi(1)$ is the unit element of R' (15 Marks)
8. Do the following sets form integral domains with respect to ordinary addition and multiplication? If so, state if they are fields: (5+6+4=15 Marks)
(i) The set of numbers of the form $b\sqrt{2}$ with b rational.
(ii) The set of even integers.
(iii) The set of positive integers.

2014

9. Let G be the set of all real 2×2 $\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$ where $xz \neq 0$ matrices where. Show that G is group under matrix multiplication. Let N denote the subset $\left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R \right\}$. Is N a normal subgroup of G ? Justify your answer. (10 Marks)

10. Show that Z_7 is a field. Then find $([5] + [6])^{-1}$ and $(-[4])^{-1}$ in Z_7 **(15 Marks)**
11. Show that the set $\{a + b\omega : \omega^3 = 1\}$, where a and b are real numbers, is a field with respect to usual addition and multiplication. **(15 Marks)**
12. Prove that the set $Q(\sqrt{5}) = \{a + b\sqrt{5} : a, b \in Q\}$ is commutative ring with identity. **(15 Marks)**

2013

13. Show that the set of matrices $S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \in R \right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f : C \rightarrow S$ defined by $f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here R is the set of real numbers and C is the set of complex numbers) **(10 Marks)**
14. Give an example of an infinite group in which every element has finite order **(10 Marks)**
15. What are the orders of the following permutation in S_{10} ? $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$ and $(1 \ 2 \ 3 \ 4 \ 5)(6 \ 7)$ **(10 Marks)**
16. What is the maximal possible order of an element in S_{10} ? Why? Give an example of such an element. How many elements will there be in S_{10} of that order? **(13 Marks)**
17. Let $J = \{a + ib \mid a, b \in Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J : Euclidean domain, principal ideal domain, and unique factorization domain? Justify your answer **(15 Marks)**
18. Let $R^C =$ ring of all real value continuous functions on $[0, 1]$, under the operations $(f + g)x = f(x) + g(x)$, $(fg)x = f(x)g(x)$. Let $M = \left\{ f \in R^C \mid f\left(\frac{1}{2}\right) = 0 \right\}$. Is M a maximal ideal of R ? Justify your answer. **(15 Marks)**

2012

19. How many elements of order 2 are there in the group of order 16 generated by a and b such that the order of a is 8, the order of b is 2 and $bab^{-1} = a^{-1}$. **(12 Marks)**
20. How many conjugacy classes does the permutation group S_5 of permutation 5 numbers have? Write down one element in each class (preferably in terms of cycles). **(15 Marks)**
21. Is the ideal generated by 2 and X in the polynomial ring $Z[X]$ of polynomials in a single variable X with coefficients in the ring of integers Z , a principal ideal? Justify your answer **(15 Marks)**
22. Describe the maximal ideals in the ring of Gaussian integers $Z[i] = \{a + ib \mid a, b \in Z\}$. **(20 Marks)**

2011

23. Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by $f_1(z) = z, f_2(z) = 1 - z, f_3(z) = \frac{z}{1-z}, f_4(z) = \frac{1}{z}, f_5(z) = \frac{1}{1-z}, f_6(z) = \frac{z-1}{z}$ is a non-abelian group of order 6 w.r.t. composition of mappings **(12 Marks)**
24. Prove that a group of Prime order is abelian. **(6 Marks)**
25. How many generators are there of the cyclic group (G, \cdot) of order 8? **(6 Marks)**
26. Give an example of a group G in which every proper subgroup is cyclic but the group itself is not cyclic **(15 Marks)**
27. Let F be the set of all real valued continuous functions defined on the closed interval $[0, 1]$. Prove that $(F, +, \cdot)$ is a Commutative Ring with unity with respect to addition and multiplication of functions defined point wise as below:

$$\left. \begin{aligned} (f+g)x &= f(x) + g(x) \\ \text{and } (fg)x &= f(x)g(x) \end{aligned} \right\} x \in [0, 1] \text{ where } f, g \in F$$
 (15 Marks)
28. Let a and b be elements of a group, with $a^2 = e, b^6 = e$ and $ab = b^4a$. Find the order of ab , and express its inverse in each of the forms $a^m b^n$ and $b^m a^n$ **(20 Marks)**

2010

29. Let $G = R - \{-1\}$ be the set of all real numbers omitting -1. Define the binary relation $*$ on G by $a * b = a + b + ab$. Show $(G, *)$ is a group and it is abelian **(12 Marks)**
30. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify. **(12 Marks)**
31. Let (R^*, \cdot) be the multiplicative group of non-zero reals and $(GL(n, R), X)$ be the multiplicative group of $n \times n$ non-singular real matrices. Show that the quotient group $\frac{GL(n, R)}{SL(n, R)}$ and (R^*, \cdot) are isomorphic where $SL(n, R) = \{A \in GL(n, R) / \det A = 1\}$ what is the center of $GL(n, R)$ **(15 Marks)**
32. Let $C = \{f : I = [0, 1] \rightarrow R / f \text{ is continuous}\}$. Show C is a commutative ring with 1 under point wise addition and multiplication. Determine whether C is an integral domain. Explain. **(15 Marks)**
33. Consider the polynomial ring $Q[x]$. Show $p(x) = x^3 - 2$ is irreducible over Q . Let I be the ideal $Q[x]$ in generated by $p(x)$. Then show that $\frac{Q[x]}{I}$ is a field and that each element of it is of the form $a_0 + a_1 t + a_2 t^2$ with a_0, a_1, a_2 in Q and $t = x + I$ **(15 Marks)**
34. Show that the quotient ring $\frac{Z[i]}{1+3i}$ is isomorphic to the ring $\frac{Z}{10Z}$ where $Z[i]$ denotes the ring of Gaussian integers **(15 Marks)**

2009

35. If R is the set of real numbers and R_+ is the set of positive real numbers, show that R under addition $(R, +)$ and R_+ under multiplication (R_+, \cdot) are isomorphic. Similarly if Q is set of rational numbers and Q_+ is the set of positive rational numbers, are $(Q, +)$ and (Q_+, \cdot) isomorphic? Justify your answer. **(4+8=12 Marks)**

36. Determine the number of homomorphisms from the additive group Z_{15} to the additive group Z_{10} (Z_n is the cyclic group of order n) **(12 Marks)**
37. How many proper, non-zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $Z_{12} \oplus Z_{12}$ have? Why? **(2+3+4+6=15Marks)**
38. Show that the alternating group of four letters A_4 has no subgroup of order 6. **(15 Marks)**
39. Show that $Z[X]$ is a unique factorization domain that is not a principal ideal domain (Z is the ring of integers). Is it possible to give an example of principal ideal domain that is not a unique factorization domain? ($Z[X]$ is the ring of polynomials in the variable X with integer.) **(15 Marks)**
40. How many elements does the quotient ring $\frac{Z_5[X]}{X^2+1}$ have? Is it an integral domain? Justify yours answers. **(15 Marks)**

2008

41. Let R_0 be the set of all real numbers except zero. Define a binary operation $*$ on R_0 as $a * b = |a|b$ where $|a|$ denotes absolute value of a . Does $(R_0, *)$ form a group? Examine. **(12 Marks)**
42. Suppose that there is a positive even integer n such that $a^n = a$ for all the elements a of some ring R . Show that $a + a = 0$ for all $a \in R$ and $a + b = 0 \Rightarrow a = b$ for all $a, b \in R$ **(12 Marks)**
43. Let G and \bar{G} be two groups and let $\phi: G \rightarrow \bar{G}$ be a homomorphism. For any element $a \in G$
- Prove that $O(\phi(a))/O(a)$
 - $\text{Ker } \phi$ is normal subgroup of G . **(15 Marks)**
44. Let R be a ring with unity. If the product of any two non-zero elements is non-zero. Then prove that $ab = 1 \Rightarrow ba = 1$. Whether Z_6 has the above property or not explain. Is Z_6 an integral domain? **(15 Marks)**
45. Prove that every Integral Domain can be embedded in a field. **(15 Marks)**
46. Show that any maximal ideal in the commutative ring $F[x]$ of polynomial over a field F is the principal ideal generated by an irreducible polynomial. **(15 Marks)**

2007

47. If in a group G , $a^5 = e$, e is the identity element of G $aba^{-1} = b^2$ for $a, b \in G$, then find the order of b **(12 Marks)**
48. Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in Z$. Show that R is a ring under matrix addition and multiplication $\left\{ A = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}, a, b \in Z \right\}$. Then show that A is a left ideal of R but not a right ideal of R . **(12 Marks)**
49. (i) Prove that there exists no simple group of order 48. **(15 Marks)**
(ii) $1 + \sqrt{-3}$ and $Z[\sqrt{-3}]$ is an irreducible element, but not prime. Justify your answer. **(15 Marks)**
50. Show that in the ring $R = \{a + b\sqrt{-5}/a, b \in Z\}$. The element $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R , where $\gamma = 7(1 + 2\sqrt{-5})$ **(30 Marks)**

2006

51. Let S be the set of all real numbers except -1 . Define on S by $a * b = a + b + ab$. Is $(S, *)$ a group? Find the solution of the equation $2 * x * 3 = 7$ in S . (12 Marks)
52. If G is a group of real numbers under addition and N is the subgroup of G consisting of integers, prove that $\frac{G}{N}$ is isomorphic to the group H of all complex numbers of absolute value 1 under multiplication (12 Marks)
53. (i) Let $O(G) = 108$. Show that there exists a normal subgroup of order 27 or 9. (10 Marks)
(ii) Let G be the set of all those ordered pairs (a, b) of real numbers for which $a \neq 0$ and define in G , an operation as follows: $(a, b) \otimes (c, d) = (ac, bc + d)$ Examine whether G is a group w.r.t the operation \otimes . If it is a group, is G abelian? (10 Marks)
54. Show that $Z[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in Z\}$ is a Euclidean domain. (30 Marks)

2005

55. If M and N are normal subgroups of a group G such that $M \cap N = \{e\}$, show that every element of M commutes with every element of N . (12 Marks)
56. Show that $(1 + i)$ is a prime element in the ring R of Gaussian integers. (12 Marks)
57. Let H and K be two subgroups of a finite group G such that $|H| > \sqrt{|G|}$ and $|K| > \sqrt{|G|}$. Prove that $H \cap K \neq \{e\}$. (15 Marks)
58. If $f : G \rightarrow G'$ is an isomorphism, prove that the order of $a \in G$ is equal to the order of $f(a)$ (15 Marks)
59. Prove that any polynomial ring $F[x]$ over a field F is U.F.D (30 Marks)

2004

60. If p is prime number of the form $4n + 1$, n being a natural number, then show that congruence $x^2 \equiv -1 \pmod{p}$ is solvable. (12 Marks)
61. Let G be a group such that of all $a, b \in G$ (i) $ab = ba$ (ii) $(O(a), O(b)) = 1$ then show that $O(ab) = O(a) O(b)$ (12 Marks)
62. Verify that the set E of the four roots of $x^4 - 1 = 0$ forms a multiplicative group. Also prove that a transformation $T, T(n) = i^n$ is a homomorphism from I_+ (Group of all integers with addition) onto E under multiplication. (10 Marks)
63. Prove that if cancellation law holds for a ring R then $a (\neq 0) \in R$ is not a zero divisor and conversely (10 Marks)
64. The residue class ring $\frac{Z}{(m)}$ is a field iff m is a prime integer. (15 Marks)
65. Define irreducible element and prime element in an integral domain D with units. Prove that every prime element in D is irreducible and converse of this is not (in general) true. (25 Marks)

2003

66. If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G (12 Marks)
67. Show that the ring $Z[i] = \{a + ib/\alpha, b \in Z, i = \sqrt{-1}\}$ of Gaussian integers is a Euclidean domain (12 Marks)
68. Let R be the ring of all real-valued continuous functions on the closed interval $[0, 1]$.
Let $M = \left\{f(x) \in R / f\left(\frac{1}{3}\right) = 0\right\}$. Show that M is a maximal ideal of R (10 Marks)
69. Let M and N be two ideals of a ring R . Show that $M \cup N$ is an ideal of R if and only if either $M \subseteq N$ or $N \subseteq M$ (10 Marks)
70. Show that $Q(\sqrt{3}, i)$ is a splitting field for $x^5 - 3x^3 + x^2 - 3$ where Q is the field of rational numbers (15 Marks)
71. Prove that $x^2 + x + 4$ is irreducible over F the field of integers modulo 11 and prove further that $\frac{F[x]}{(x^2 + x + 4)}$ is a field having 121 elements. (15 Marks)
72. Let R be a unique factorization domain (U.F.D), then prove that $R[x]$ is also U.F.D (10 Marks)

2002

73. Show that a group of order 35 is cyclic. (12 Marks)
74. Show that polynomial $25x^4 + 9x^3 + 3x + 3$ is irreducible over the field of rational numbers (12 Marks)
75. Show that a group of p^2 is abelian, where p is a prime number. (10 Marks)
76. Prove that a group of order 42 has a normal subgroup of order 7. (10 Marks)
77. Prove that in the ring $F[x]$ of polynomial over a field F , the ideal $1 = |p(x)|$ is maximal if and only if the polynomial $p(x)$ is irreducible over F . (20 Marks)
78. Show that every finite integral domain is a field (10 Marks)
79. Let F be a field with q elements. Let E be a finite extension of degree n over F . Show that E has q^n elements (10 Marks)

2001

80. Let K be a field and G be a finite subgroup of the multiplicative group of non-zero elements of K . Show that G is a cyclic group. (12 Marks)
81. Prove that the polynomial $1 + x + x^2 + x^3 + \dots + x^{p-1}$ where p is prime number is irreducible over the field of rational numbers. (12 Marks)
82. Let N be a normal subgroup of a group G . Show that $\frac{G}{N}$ is abelian if and only if for all $x, y \in G, xyz^{-1} \in N$ (20 Marks)
83. If R is a commutative ring with unit element and M is an ideal of R , then show that maximal ideal of R if and only if $\frac{R}{M}$ is a field (20 Marks)
84. Prove that every finite extension of a field is an algebraic extension. Give an example to show that the converse is not true. (20 Marks)

2000

85. Let n be a fixed positive integer and let Z_n be the ring of integers modulo n . Let $G = \{\alpha \in Z_n \mid \alpha \neq 0\}$ and a is relatively prime to n . Show that G is a group under multiplication defined in Z_n . Hence, or otherwise, show that $a^{\phi(n)} \equiv a \pmod{n}$ for all integers a relatively prime to n where $\phi(n)$ denotes the number of positive integers that are less than n and are relatively prime to n . **(20 Marks)**
86. Let M be a subgroup and N a normal subgroup of group G . Show that MN is a subgroup of G and $\frac{MN}{N}$ is isomorphic to $\frac{M}{M \cap N}$. **(20 Marks)**
87. Let F be a finite field. Show that the characteristic of F must be a prime integer p and the number of elements in F must be p^m for some positive integer m . **(20 Marks)**
88. Let F be a field and $F[x]$ denote the set of all polynomials defined over F . If $f(x)$ is an irreducible polynomial in $F[x]$, show that the ideal generated by $f(x)$ in $F[x]$ is maximal and $\frac{F[x]}{f(x)}$ is a field. **(20 Marks)**
89. Show that any finite commutative ring with no zero divisors must be a field. **(20 Marks)**

1999

90. If ϕ is a homomorphism of G into \bar{G} with kernel K , then show that K is a normal subgroup of G . **(20 Marks)**
91. If p is prime number and $p^\alpha \mid O(G)$, then prove that G has a subgroup of order p^α . **(20 Marks)**
92. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field. **(20 Marks)**

1998

93. Prove that if a group has only four elements then it must be abelian. **(20 Marks)**
94. If H and K are subgroups of a group G then show that HK is a subgroup of G if and only if $HK = KH$. **(20 Marks)**
95. Let $(R, +, \cdot)$ be a system satisfying all the axioms for a ring with unity with the possible exception of $a + b = b + a$. Prove that $(R, +, \cdot)$ is a ring. **(20 Marks)**
96. If p is prime then prove that Z_p is a field. Discuss the case when p is not a prime number. **(20 Marks)**
97. Let D be a principal domain. Show that every element that is neither zero nor a unit in D is a product of irreducibles. **(20 Marks)**

1997

98. Show that a necessary and sufficient condition for a subset H of a group G to be a subgroup is $HH^{-1} = H$. **(20 Marks)**
99. Show that the order of each subgroup of a finite group is a divisor of the order of the group. **(20 Marks)**

100. In a group G , the commutator (a, b) $a, b \in G$ is the element $aba^{-1}b^{-1}$ and the smallest subgroup containing all commutators is called the commutator subgroup of G . Show that a quotient group $\frac{G}{H}$ is abelian if and only if H contains the commutator subgroup of G . **(20 Marks)**
101. If $x^2 = x$ for all x in a ring R , show that R is commutative. Give an example to show that the converse is not true. **(20 Marks)**
102. Show that an ideal S of the ring of integers Z is maximal ideal if and only if S is generated by a prime integer. **(20 Marks)**
103. Show that in an integral domain every prime element is irreducible. Give an example to show that the converse is not true. **(20 Marks)**

1996

104. Let R be the set of real numbers and $G = \{(a, b) \mid a, b \in R, a \neq 0\}$. $G \times G \rightarrow G$ is defined by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a group. Is it abelian? **(20 Marks)**
105. Let f be a homomorphism of a group G onto a group G' with kernel H . For each subgroup K' of G' define K by. Prove that
- (i) K' is isomorphic to $\frac{K}{H}$
- (ii) $\frac{G}{K}$ is isomorphic to $\frac{G'}{K'}$ **(20 Marks)**
106. Prove that a normal subgroup H of a group G is maximal, if and only if the quotient group $\frac{G}{H}$ is simple. **(20 Marks)**
107. In a ring R , prove that cancellation laws hold. If and only if R has no zero divisors. **(20 Marks)**
108. If S is an ideal of ring R and T any subring of R , then prove that S is an ideal of $S + T = \{s + t \mid s \in S, t \in T\}$. **(20 Marks)**
109. Prove that the polynomial $x^2 + x + 4$ is irreducible over the field of integers modulo 11. **(20 Marks)**

1995

110. Let G be a finite set closed under an associative binary operation such that $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$. Prove that G is a group. **(20 Marks)**
111. Let G be group of order p^n , where p is a prime number and $n > 0$. Let H be a proper subgroup of G and $N(H) = \{x \in G : x^{-1}hx \in H \forall h \in H\}$. Prove that $N(H) \neq H$. **(20 Marks)**
112. Show that a group of order 112 is not simple. **(20 Marks)**
113. Let R be a ring with identity. Suppose there is an element a of R which has more than one right inverse. Prove that a has infinitely many right inverses. **(20 Marks)**
114. Let F be a field and let $p(x)$ be an irreducible polynomial over F . Let $\langle p(x) \rangle$ be the ideal generated by $p(x)$. Prove that $\langle p(x) \rangle$ is a maximal ideal. **(20 Marks)**
115. Let F be a field of characteristic $p \neq 0$. Let $F(x)$ be the polynomial ring. Suppose $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is an element of $F(x)$. Define $f(x) = a_1 + 2a_2x + 3a_3x^2 + \dots + na_nx^{n-1}$. If $f(x) = 0$, prove that there exists $g(x) \in F(x)$ such that $f(x) = g(x)^p$. **(20 Marks)**

1994

116. If G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers n for all $a, b \in G$, then prove that G is abelian. (20 Marks)
117. Can a group of order 42 be simple? Justify your claim (20 Marks)
118. Show that the additive group of integers modulo 4. Is isomorphic to the multiplicative group of the non-zero elements of integers modulo 5. State the two isomorphisms (20 Marks)
119. Find all the units of the integral domain of Gaussian integers. (20 Marks)
120. Prove or disprove the statement: The polynomial ring $I[x]$ over the ring of integers is a principal ideal ring. (20 Marks)
121. If R is an integral domain (not necessarily a unique factorization domain) and F is its field of quotients, then show that any element $f(x)$ in $F(x)$ is of the form $f(x) = \frac{f_0(x)}{a}$ where $f_0(x) \in R[x], a \in R$. (20 Marks)

1993

122. If G is a cyclic group of order n and p divides n , then prove that there is a homomorphism of G onto a cyclic group of order p . What is the Kernel of homomorphism? (20 Marks)
123. Show that a group of order 56 cannot be simple. (20 Marks)
124. Suppose that H, K are normal subgroups of a finite group G with H a normal subgroup of K . If $P = \frac{K}{H}, S = \frac{G}{H}$, then prove that the quotient groups $\frac{S}{P}$ and $\frac{G}{K}$ are isomorphic. (20 Marks)
125. If Z is the set of integers then show that $Z[\sqrt{-3}] = \{a + \sqrt{-3}b : a, b \in Z\}$ is not a unique factorization domain (20 Marks)
126. Construct the addition and multiplication table for $\frac{Z_3[x]}{\langle x^2 + 1 \rangle}$ where Z_3 is the set of integers modulo 3 and $\langle x^2 + 1 \rangle$ is the ideal generated by $(x^2 + 1)$ in $Z_3[x]$ (20 Marks)
127. Let Q be the set of rational number and $Q(2^{1/2}, 2^{1/3})$ the smallest extension field of Q containing $2^{1/2}, 2^{1/3}$. Find the basis for $Q(2^{1/2}, 2^{1/3})$ over Q . (20 Marks)

1992

128. If H is a cyclic normal subgroup of a group G , then show that every subgroup of H is normal in G . (20 Marks)
129. Show that no group of order 30 is simple. (20 Marks)
130. If p is the smallest prime factor of the order of a finite group G , prove that any subgroup of index p is normal. (20 Marks)
131. If R is unique factorization domain, then prove that any $f \in R[x]$ is an irreducible element of $R[x]$, if and only if either f is an irreducible element of R or f is an irreducible polynomial in $R[x]$. (20 Marks)

132. Prove that $x^2 + 1$ and $x^2 + x + 4$ are irreducible over F , the field of integers modulo 11. Prove also that $\frac{F[x]}{\langle x^2 + 1 \rangle}$ and $\frac{F[x]}{\langle x^2 + x + 4 \rangle}$ are isomorphic fields each having 121 elements. **(20 Marks)**
133. Find the degree of splitting field $x^5 - 3x^3 + x^2 - 3$ over Q , the field of rational numbers. **(20 Marks)**

RAJANVASRI