RAMANASRI

IAS/IFoS Institute

# Ordinary Differential Equations (ODE)

## Previous year Questions from 2017 to 1992

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# 2017

1. Find the differential equation representing the entire circle in the $xy$- plane. **[10 Marks]**
2. Solve the following simultaneous liner differential equations: $(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where $y$ and $z$ are functions of independent variable $x$ and $D \equiv \frac{d}{dx}$. **[8 Marks]**
3. If the growth rate of the population of bacteria at time $t$ is proportional to the amount present at the time and population doubles in one week, then how much bacteria's can be expected after 4 weeks? **[8 Marks]**
4. Consider the differential equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \frac{dy}{dx}$ substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of $u, v$ and $p' = \frac{dv}{du}$ hence or otherwise solve the equation. **[10 Marks]**
5. Solve the following initial value differential equations $20y'' + 4y' + y = 0,\ y(0) = 3.2,\ y'(0) = 0.$ **[7 Marks]**
6. Solve the differential equation: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin(x^2)$ **[9 Marks]**
7. Solve that following differential equation using method of variation of parameters $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$. **[8 Marks]**
8. Solve the following initial value problem using Laplace transform: $\frac{d^2y}{dx^2} + 9y = r(x),\ y(0) = 0,\ y'(0) = 4$ where $r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases}$. **[17 Marks]**

# 2016

9. Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2}\sin\frac{x\sqrt{3}}{2}$ **[10 marks]**
10. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self orthogonal. **[10 marks]**
11. Solve $\{y(1 - x\tan x) + x^2\cos x\}dx - xdy = 0$ **[10 marks]**
12. Using the method of variation of parameter solve the differential equation $(D^2 + 2D + 1)y = e^{-x}\log(x),\quad \left[D \equiv \frac{d}{dx}\right]$ **[15 marks]**
13. Find the general solution of the equation $x^2\frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$ **[15 marks]**
14. Using Laplace transformation solves the following: $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$ **[10 marks]**

# 2015

15. Solve the differential equation: $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$ **[10 Marks]**
16. Solve the differential equation: $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ **[10 Marks]**

17. Find the constant $a$ so that $(x+y)^a$ is the integrating factor of $(4x^2+2xy+6y)dx+(2x^2+9y+3x)dy=0$ and hence solve the differential equation **[12 Marks]**

18. (i) Obtain Laplace Inverse transform of $\left\{\ln\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-5s}\right\}$

(ii) Using Laplace transform, solve $y''+y=t, y(0)=1, y'(0)=-2$ **[6+6=12 Marks]**

19. Solve the differential equation $x=py-p^2$ where $p=\frac{dy}{dx}$ **[13 Marks]**

20. Solve $x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x)$ **[13 Marks]**

# 2014

21. Justify that a differential equation of the form: $\left[y+xf(x^2+y^2)\right]dx+\left[yf(x^2+y^2)-x\right]dy=0$ where $f(x^2+y^2)$ is an arbitrary function of $(x^2+y^2)$, is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2)=(x^2+y^2)^2$

**[10 Marks]**

22. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency **[10 Marks]**

23. Solve by the method of variation of parameters: $\frac{dy}{dx}-5y=\sin x$ **(10 Marks]**

24. Solve the differential equation: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos\left(\log_e x\right)$ **[20 Marks]**

25. Solve the following differential equation: $x\frac{d^2y}{dx^2}-2\left(x+1\right)\frac{dy}{dx}+(x+2)y=\left(x-2\right)e^{2x}$, when $e^x$ is a solution to its corresponding homogeneous differential equation. **[15 Marks]**

26. Find the sufficient condition for the differential equation $M\left(x,y\right)dx+N\left(x,y\right)dy=0$, to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation of $(x^2+xy)dx+(y^2+xy)dy=0$ and solve it.

**[15 Marks]**

27. Solve the initial value problem $\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t,\ y(0)=0, y'(0)=0$ by using Laplace transform.

**[20 Marks]**

# 2013

28. If $y$ is a function of $x$, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos\left(x+y\right)+\sin\left(x+y\right)$. Find out a relation between $x$ and $y$, which is free from any derivative / differential. **[10 Marks]**

29. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n=a\sin n\theta,\ (r,\theta)$ being the plane polar coordinates. **[10 Marks]**

30. Solve the differential equation $(5x^3+12x^2+6y^2)dx+6xydy=0$ **[15 Marks]**

31. Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec ax$

**[15 Marks]**

32. Find the general solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\ln x\sin(\ln x)$ **[15 Marks]**

33. By using Laplace transform method, solve the differential equation $(D^2+n^2)x=a\sin(nt+\alpha)$, $D^2=\frac{d^2}{dt^2}$ subject to the initial conditions $x=0$ and $\frac{dx}{dt}=0$, at $t=0$, in which $a, n$ and $\alpha$ are constants. **[15 Marks]**

# 2012

34. Solve $\frac{dy}{dx}=\frac{2xye^{(x/y)^2}}{y^2(1+e^{(x/y)^2})+2x^2e^{(x/y)^2}}$ **[12 Marks]**

35. Find the orthogonal trajectory of the family of curves $x^2+y^2=ax$ **[12 Marks]**

36. Using Laplace transforms, solve the initial value problem $y''+2y'+y=e^{-t}$, $y(0)=-1, y'(0)=1$

**[12 Marks]**

37. Show that the differential equation $(2xy\log y)dx+(x^2+y^2\sqrt{y^2+1})dy=0$ is not exact. Find an integrating factor and hence, the solution of the equation **[20 Marks]**

38. Find the general solution of the equation $y'''-y''=12x^2+6x$ **[20 Marks]**

39. Solve the ordinary differential equation $x(x-1)y''-(2x-1)y'+2y=x^2(2x-3)$ **[20 Marks]**

# 2011

40. Obtain the solution of the ordinary differential equation $\frac{dy}{dx}=\left(4x+y+1\right)^2$, if $y(0)=1$ **[10 Marks]**

41. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r=a(1-\cos\theta)$, $(r,\theta)$ being the plane polar coordinates of any point. **[10 Marks]**

42. Obtain Clairaut's form of the differential equation $\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+x\right)=a^2\frac{dy}{dx}$. Also find its general solution **[15 Marks]**

43. Obtain the general solution of the second order ordinary differential equation $y''-2y'+2y=x+e^x\cos x$, where dashes denote derivatives w.r.t. $x$ **[15 Marks]**

44. Using the method of variation of parameters, solve the second order differential equation $\frac{d^2y}{dx^2}+4y=\tan 2x$ **[15 Marks]**

45. Use Laplace transform method to solve the following initial value problem: $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$, $x(0)=2$ and $\left.\frac{dy}{dt}\right|_{t=0}=-1$ **[15 Marks]**

# 2010

46. Consider the differential equation $y' = \alpha x,\ x > 0$ where $\alpha$ is a constant. Show that
  (i) If $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
  (ii) If $\alpha < 0$, then every solution tends to zero as $x \to \infty$ **[12 Marks]**

47. Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation **[12 Marks]**

48. Verify that $\frac{1}{2}(Mx + Ny)d\left[\log_e(xy)\right] + \frac{1}{2}(Mx - Ny)d\left[\log_e(x/y)\right] = Mdx + Ndy$. Hence show that-
  (i) If the differential equation $Mdx + Ndy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;
  (ii) If the differential equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx + Ny \equiv 0$; **[20 Marks]**

49. Use the method of undermined coefficients to find the particular solutions of $y'' + y = \sin x + (1 + x^2)e^x$ and hence find its general solution. **[20 Marks]**

# 2009

50. Find the Wronskian of the set of functions: $\left\{3x^3,\ \left|3x^3\right|\right\}$ on the interval $[-1,\ 1]$ and determine whether the set is linearly dependent on $[-1,\ 1]$ **[12 Marks]**

51. Find the differential equation of the family of circles in the $xy$-plane passing through $(-1,\ 1)$ and $(1,\ 1)$ **[20 Marks]**

52. Find the inverse Laplace transform of $F(s) = 1n\left(\frac{s+1}{s+s}\right)$ **[20 Marks]**

53. Solve : $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3},\ y(0) = 1$ **[20 Marks]**

# 2008

54. Solve the differential equation $ydx + (x + x^3y^2)dy = 0$ **[12 Marks]**

55. Use the method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = -x^4 \sin x$ **[12 Marks]**

56. Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t},\ y(0) = 1,\ y'(0) = -1$ **[15 Marks]**

57. Solve the differential equation $x^3y'' - 3x^2y' + xy = \sin(\ln x) + 1$ **[15 Marks]**

58. Solve the equation $y - 2xp + yp^2 = 0$, where $p = \frac{dy}{dx}$ **[15 Marks]**

# 2007

59. Solve the ordinary differential equation $\cos 3x\dfrac{dy}{dx}-3y\sin 3x=\dfrac{1}{2}\sin 6x+\sin^2 3x, 0<x<\dfrac{\pi}{2}$

**[12 Marks]**

60. Find the solution of the equation $\dfrac{dy}{y}+xy^2dx=-4xdx$ **[12 Marks]**

61. Determine the general and singular solutions of the equation $y=x\dfrac{dy}{dx}+a\dfrac{dy}{dx}\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$, $a$ being a constant. **[15 Marks]**

62. Obtain the general solution of $[D^3-6D^2+12D-8]y=12\left(e^{2x}+\dfrac{9}{4}e^{-x}\right)$, where $D\equiv\dfrac{dy}{dx}$

**[15 Marks]**

63. Solve the equation $2x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}-3y=x^3$ **[15 Marks]**

64. Use the method of variation of parameters to find the general solution of the equation $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=2e^x$ **[15 Marks]**

# 2006

65. Find the family of curves whose tangents form an angle $\dfrac{\pi}{4}$ with the hyperbolas $xy=c, c>0$

**[12 Marks]**

66. Solve the differential equation $\left(xy^2+e^{-\frac{1}{x^3}}\right)dx-x^2ydy=0$ **[12 Marks]**

67. Solve: $(1+y^2)+(x-e^{-\tan^{-1}y})\dfrac{dy}{dx}=0$ **[15 Marks]**

68. Solve the equation $x^2p^2+py(2x+y)+y^2=0$ using the substitution $y=u$ and $xy=v$ and find its singular solution, where $p=\dfrac{dy}{dx}$ **[15 Marks]**

69. Solve the differential equation $x^2\dfrac{d^3y}{dx^3}+2x\dfrac{d^2y}{dx^2}+2\dfrac{y}{x}=10\left(1+\dfrac{1}{x^2}\right)$ **[15 Marks]**

70. Solve the differential equation $(D^2-2D+2)y=e^x\tan x$, $D\equiv\dfrac{dy}{dx}$ by the method of variation of parameters. **[15 Marks]**

# 2005

71. Find the orthogonal trajectory of the family of co-axial circles $x^2+y^2+2gx+c=0$, where $g$ is the parameter. **[12 Marks]**

72. Solve: $xy\frac{dy}{dx}=\sqrt{(x^2-y^2-x^2y^2-1)}$ **[12 Marks]**

73. Solve the differential equation: $\left[(x+1)^4D^3+2(x+1)^3D^2-(x+1)^2D+(x+1)\right]y=\frac{1}{(x+1)}$

**[15 Marks]**

74. Solve the differential equation: $(x^2+y^2)(1+p)^2-2(x+y)(1+p)(x+yp)+(x+yp)^2=0$ where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **[15 Marks]**

75. Solve the differential equation $(\sin x-x\cos x)y''-x\sin x\ y'+y\sin x=0$ given that $y=\sin x$ is a solution of this equation. **[15 Marks]**

76. Solve the differential equation $x^2y''-2xy'+2y=x\log x,\ x>0$ by variation of parameters

**[15 Marks]**

# 2004

77. Find the solution of the following differential equation $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$ **[12 Marks]**

78. Solve: $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ **[12 Marks]**

79. Solve: $(D^4-4D^2-5)y=e^x(x+\cos x)$ **[15 Marks]**

80. Reduce the equation $(px-y)(py+x)=2p$, where $p=\frac{dy}{dx}$ to Clairaut's equation and hence solve it. **[15 Marks]**

81. Solve: $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$ **[15 Marks]**

82. Solve the following differential equation: $(1-x^2)\frac{d^2y}{dx^2}-4x\frac{dy}{dx}-(1+x^2)y=x$ **[15 Marks]**

# 2003

83. Show that the orthogonal trajectory of a system of confocals ellipses is self orthogonal **[12 Marks]**

84. Solve: $x\frac{dy}{dx}+y\log y=xye^x$ **[12 Marks]**

85. Solve $(D^5-D)=4(e^x+\cos x+x^3)$, where $D\equiv\frac{dy}{dx}$ . **[15 Marks]**

86. Solve the differential equation $(px^2+y^2)(px+y)=(P+1)^2$, where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions **[15 Marks]**

87. Solve $(1-x^2)y''+(1+x)y'+y=\sin 2\left[\log(1+x)\right]$ **[15 Marks]**

88. Solve the differential equation $x^2y''-4x\ y'+6y=x^4\sec^2 x$ by variation of parameters. **[15 Marks]**

# 2002

89. Solve : $x\frac{dy}{dx}+3y=x^3y^2$ **[12 Marks]**

90. Find the values of $\lambda$ for which all solutions of $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x\to\infty$. **[12 Marks]**

91. Find the value of constant $\lambda$ such that the following differential equation becomes exact. $(2xe^y+3y^2)\frac{dy}{dx}+(3x^2+\lambda e^y)=0$. Further, for this value of $\lambda$, solve the equation. **[15 Marks]**

92. Solve : $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$ **[15 Marks]**

93. Using the method of variation of parameters, find the solutions of $\frac{d^2y}{d^2x}-2\frac{dy}{dx}+y=xe^x\sin x$ with $y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$ **[15 Marks]**

94. Solve : $(D-1)(D^2-2D+2)y=e^x$ where $D\equiv\frac{dy}{dx}$ **[15 Marks]**

# 2001

95. A continuous function $y(t)$ satisfies the differential equation $\frac{dy}{dx}=\begin{cases}1+e^{1-t}, & 0\le t<1\\ 2+2t-3t^2, & 1\le t<5\end{cases}$ If $y(0)=-e$ find $y(2)$ **[12 Marks]**

96. Solve : $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log_e x$ **[12 Marks]**

97. Solve : $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$ **[15 Marks]**

98. Find the general solution of $ayp^2+(2x-b)p-y=0,\ a>0$ **[15 Marks]**

99. Solve: $(D^2+1)^2y=24x\cos x$ given that $y=Dy=D^2y=0$ and $D^3y=12$ when $x=0$ **[15 Marks]**

100. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$ **[15 Marks]**

# 2000

101. Show that $3\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-8y=0$ has an integral which is a polynomial in $x$. Deduce the general solution. **[12 Marks]**

102. Reduce $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, where $P,Q,R$ are functions of $x$, to the normal form. Hence solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$ **[15 Marks]**

103. Solve the differential equation $y = x - 2ap + ap^2$. Find the singular solution and interpret it geometrically **[15 Marks]**

104. Show that $(4x+3y+1)dx+(3x+2y+1)dy=0$ represents a family of hyperbolas with a common axis and tangent at the vertex **[15 Marks]**

105. Solve $x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right)$ by the method of parameters **[15 Marks]**

# 1999

106. Solve the differential equation $\frac{xdx+ydy}{xdy-ydx} = \left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$ **[20 Marks]**

107. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$ **[20 Marks]**

108. By the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec(ax)$ **[20 Marks]**

# 1998

109. Solve the differential equation: $xy - \left(\frac{dy}{dx}\right) = y^3e^{-x^2}$ **[20 Marks]**

110. Show that the equation: $(4x+3y+1)dx+(3x+2y+1)dy=0$ represents a family of hyperbolas having as asymptotes the lines $x+y=0,\ 2x+y+1=0$ . **[20 Marks]**

111. Solve the differential equation: $y = 3px + 4p^2$ **[20 Marks]**

112. Solve the differential equation: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}(x^2+9)$ **[20 Marks]**

113. Solve the differential equation: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\sin x$ **[20 Marks]**

114. Solve the differential equation: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **[20 Marks]**

# 1997

115. Solve the initial value problem $\frac{dy}{dx} = \frac{x}{x^2y+y^3}$, $y(0)=0$ **[20 Marks]**

116. Solve $(x^2-y^2+3x-y)dx+(x^2-y^2+x-3y)dy=0$ **[20 Marks]**

117. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm. find an expression for the radius of the rain drop at any time. **[20 Marks]**

118. Solve $\frac{d^4y}{dx^4} + 6\frac{d^3y}{dx^3} + 11\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 20e^{-2x}\sin x$ **[20 Marks]**

119. Make use of the transformation $y(x) = u(x)\sec x$ to obtain the solution of
$y'' - 2y'\tan x + 5y = 0,\ y(0) = 0,\ y'(0) = \sqrt{6}$ **[20 Marks]**

120. Solve $(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2,\ y(0) = 0,\ y'(0) = 2$ **[20 Marks]**

# 1996

121. Find the curves for which the sum of the reciprocals of the radius vector and polar sub tangent is constant. **[20 Marks]**

122. Solve : $x^2(y - px) = yp^2,\ p \equiv \frac{dy}{dx}$ **[20 Marks]**

123. Solve : $y\sin 2x dx - \left(1 + y^2 + \cos^2 x\right)dy = 0$ **[20 Marks]**

124. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$. Find the value of $y$ when $x = \frac{\pi}{2}$, if it is given that $y = 3$ and $\frac{dy}{dx} = 0$ when $x = 0$ **[20 Marks]**

125. Solve : $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} = x^2 + 3e^{2x} + 4\sin x$ **[20 Marks]**

126. Solve : $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$ **[20 Marks]**

# 1995

127. Determine a family of curves for which the ratio of the $y$- intercept of the tangent to the radius vector is a constant. **[20 Marks]**

128. Solve $(2x^2 + 3y^2 - 7)xdx + (3x^2 + 2y^2 - 8)ydy = 0$ **[20 Marks]**

129. Test whether the equation $(x+y)^2 dx - (y^2 - 2xy - x^2)dy = 0$ is exact and hence solve it. **[20 Marks]**

130. Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **[20 Marks]**

131. Determine all real valued solutions of the equations $y''' - iy'' + y' - iy = 0,\ y' = \frac{dy}{dx}$ **[20 Marks]**

132. Find the solution of the equation $\frac{d^2y}{dx^2} + 4y = 8\cos 2x$, given that $y = 0$ and $y' = 2$ when $x = 0$ **[20 Marks]**

# 1994

133. Solve: $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ **[20 Marks]**

134. Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ is a function of $x$ only, say , $f(x)$, then $F(x) = e^{\int f(x)dx}$ is an integration factor of $Pdx + Qdy = 0$ **[20 Marks]**

135. Find the family of curves whose tangent from an angel $\frac{\pi}{4}$ with the hyperbola $xy = c$ **[20 Marks]**

136. Transform the differential equation $\frac{d^2y}{dx^2}\cos x + \frac{dy}{dx}\sin x - 2y\cos^3 x = 2\cos^5 x$ into one having $z$ as independent variable where $z = \sin x$ and solve it. **[20 Marks]**

137. If $\frac{d^2x}{dt^2} + \frac{g}{b}(x - a) = 0$ ( $a, b$ and $g$ being positive constants) and $x = a'$ and $\frac{dx}{dt} = 0$ when $t = 0$, show that $x = a + (a' - a)\cos t\sqrt{\frac{g}{b}t}$ **[20 Marks]**

138. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$ where $D \equiv \frac{dy}{dx}$ **[20 Marks]**

# 1993

139. Determine the curvature for which the radius of curvature is proportional to the slope of the tangent. **[20 Marks]**

140. Show that the system of co focal conics $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self orthogonal. **[20 Marks]**

141. Solve $\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\}dx + \left(x + \log x - x\sin y\right)dy = 0$ **[20 Marks]**

142. Solve $y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 = y^2$ **[20 Marks]**

143. Solve $\frac{d^2y}{dx^2} + \omega_0{}^2 y = a\cos\omega t$ and discuss the nature of solution as $\omega \xrightarrow{dt^2} \omega_0$ **[20 Marks]**

144. Solve $(D^4 + D^2 + 1)y = e^{-x/2}\cos\left(x\frac{\sqrt{3}}{2}\right)$ **[20 Marks]**

# 1992

145. By eliminating the constants $a, b$ obtain the differential equation for which $xy = ae^x + be^{-x} + x^2$ is a solution **[20 Marks]**

146. Find the orthogonal trajectory of the family of semi cubical parabolas $ay^2 = x^3$, where $a$ is a variable parameter. **[20 Marks]**

147. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents hyperbolas having the following lines as asymptotes $x + y = 0,\ 2x + y + 1 = 0$ **[20 Marks]**

148. Solve the following differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$ **[20 Marks]**

149. Find the curves for which the portion of $y$- axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact. **[20 Marks]**

150. Solve the following differential equation: $(D^2 + 4)y = \sin 2x,$ given that when $x = 0,$ then $y = 0$ and $\frac{dy}{dx} = 2$ **[20 Marks]**

151. Solve : $(D^3 - 1)y = xe^x + \cos^2 x$ **[20 Marks]**

152. Solve : $(x^2D^2 + xD - 4)y = x^2$ **[20 Marks]**