RAMANASRI
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# Partial Differential Equations

## Previous year Questions from 2017 to 1992

*Ramanasri Institute*

2017

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# 2017

1. Solve $(D^2 - 2DD' - D'^2)z = e^{x+2y} + x^3 + \sin 2x$ where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},\ D^2 \equiv \frac{\partial^2}{\partial x^2},\ D'^2 \equiv \frac{\partial^2}{\partial y^2}.$ **[10 Marks]**

2. Let $\Gamma$ be a closed curve in $xy$-plane and let $S$ denote the region bounded by the curve $\Gamma.$ Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x,y)\ \ \forall (x,y) \in S.$ If $f$ is prescribed at each point $(x,y)$ of $S$ and $w$ is prescribed on the boundary $\Gamma$ of $S$ then prove that any solution $w = w(x,y),$ satisfying these conditions, is unique. **[10 Marks]**

3. Find a complete integral of the partial differential equation $2(pq + yp + qx) + x^2 + y^2 = 0$. **[15 Marks]**

4. Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it. **[15 Marks]**

5. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2};\ t > 0,$ where $c^2 = \frac{T}{m},\ T$ the constant tension in the string and $m$ is the mass per unit length of the string.
   (i) Find the appropriate solution of the wave equation
   (ii) Find also the solution under the conditions $y(0,t) = 0,\ y(l,t) = 0$ for all $t$ and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0,\ y(x,0) = a\sin\frac{\pi x}{t}, 0 < x < l,\ a > 0$ . **[20 Marks]**

# 2016

6. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz.$ **(10 marks)**

7. Final he general integral of the particle differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$ **(10 marks)**

8. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$ **(15 marks)**

9. Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y}\frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$ **(15 marks)**

10. Find the temperature $u(x,t)$ in a bar of silver of length and constant cross section of area $1cm^2$ . Let density $p = 10.6g/cm^3$, thermal conductivity $K = 1.04/(cm\sec^\circ C)$ and specific heat $\sigma = 0.056/g^\circ C$ the bar is perfectly isolated laterally with ends kept at $0^\circ C$ and initial temperature $f(x) = \sin(0.1\pi x)^\circ C$ note that $u(x,t)$ follows the head equation $u_t = c^2 u_{xx}$ where $c^2 = k/(\rho\sigma)$ **(20 marks)**

# 2015

11. Solve the partial differential equation: $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ **(10 Marks)**

12. Solve : $(D^2 + DD' - 2D'^2)u = e^{x+y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ **(10 Marks)**

13. Solve for the general solution $p\cos(x+y) + q\sin(x+y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ **(15 Marks)**

14. Find the solution of the initial-boundary value problem

$$u_t - u_{xx} + u = 0, \qquad 0 < x < l,\ t > 0$$
$$u(0,\ t) = u(l,\ t) = 0, \qquad t \ge 0$$
$$u(x,\ 0) = x(l - x), \qquad 0 < x < l$$

**(15 Marks)**

15. Reduce the second-order partial differential equation $x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ into canonical form. Hence, find its general solution **(15 Marks)**

# 2014

16. Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ **(10 Marks)**

17. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(15 Marks)**

18. Find the deflection of a vibrating string (length= $\pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ ) corresponding to zero initial velocity and initial deflection. $f(x) = k(\sin x - \sin 2x)$ **(15 Marks)**

19. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$, given that

(i) $u(x,\ 0) = 0, \qquad 0 \le x \le 1;$

(ii) $\frac{\partial u}{\partial t}(x,\ 0) = x^2, \quad 0 \le x \le 1$

(iii) $u(0,\ t) = u(1,\ t) = 0,\ \text{for all } t$ **(15 Marks)**

# 2013

20. From a partial differential equation by eliminating the arbitrary functions $f$ and $g$ from $z = yf(x) + xg(y)$ **(10 Marks)**

21. Reduce the equation $y\frac{\partial^2 z}{\partial x^2} + (x+y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$ to its canonical from when $x \ne y$ **(10 Marks)**

22. Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y)$ where $D$ and $D'$ denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ **(15 Marks)**

23. Find the surface which intersects the surfaces of the system $z(x+y) = C(3z+1)$, ( $C$ being a constant ) orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$ **(15 Marks)**

24. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x(l - x)$, find the displacement of the string at any distance $x$ from one end at any time $t$ **(20 Marks)**

# 2012

25. Solve partial differential equation $(D-2D')(D-D')^2 z = e^{x+y}$ **(12 Marks)**
26. Solve partial differential equation $px + qy = 3z$ **(20 Marks)**
27. A string of length $l$ is fixed at its ends. The string from the mid-point is pulled up to a height $k$ and then released from rest. Find the deflection $y(x,\ t)$ of the vibrating string. **(20 Marks)**
28. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **(20 Marks)**

# 2011

29. Solve the PDE $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2 y$ **(12 Marks)**
30. Solve the PDE $(x+2z)\frac{\partial z}{\partial x} + (4zx - y)\frac{\partial z}{\partial y} = 2x^2 + y$ **(12 Marks)**
31. Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. **(20 Marks)**
32. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 \le x \le a,\ 0 \le y \le b$ satisfying the boundary conditions
$u(0,\ y) = 0,\ u(x,\ 0) = 0,\ u(x,\ b) = 0\ \frac{\partial u}{\partial x}(a,\ y) = T\sin^3\frac{\pi y}{a}$ **(20 Marks)**
33. Obtain temperature distribution $y(x,\ t)$ in a uniform bar of unit length whose one end is kept at $10^0$ and the other end is insulated. Also it is given that $y(x,\ 0) = 1 - x,\ 0 < x < 1$ **(20 Marks)**

# 2010

34. Solve the PDE $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$ **(12 Marks)**
35. Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$ **(12 Marks)**
36. Solve the following partial differential equation

$$zp + yq = x$$
$$x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$$

by the method of characteristics. **(20 Marks)**

37. Reduce the following 2nd order partial differential equation into canonical form and find its general solution. $xu_{xx} + 2x^2 u_{xy} - u_x = 0$ **(20 Marks)**
38. Solve the following heat equation

$$u_t - u_{xx} = 0, \qquad 0 < x < 2,\ t > 0$$
$$u(0,\ t) = u(2,\ t) = 0 \quad t > 0$$
$$u(x,\ 0) = x(2-x), \qquad 0 \le x \le 2$$

**(20 Marks)**

# 2009

39. Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$. **(12 Marks)**

40. (i) Form the partial differential equation by elimination the arbitrary function $f$ given by:
$f(x^2 + y^2, z - xy) = 0$ **(20 Marks)**
(ii) Find the integral surface of: $x^2 p + y^2 p + z^2 = 0$ which passes through the curve:
$xy = x + y,\ z = 1$ **(20 Marks)**

41. Find the characteristics of: $y^2 r - x^2 t = 0$ where $r$ and $t$ have their usual meanings. **(15 Marks)**

42. Solve : $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$ where $D$ and $D'$ represent $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ **(15 Marks)**

43. A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where $\mu$ is a constant, and then released. Find the displacement of any point $x$ of the string at time $t > 0$. **(30 Marks)**

# 2008

44. Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular solution which passes through the lines $x = 1,\ y = 0$ **(12 Marks)**

45. Find the general solution of the partial differential equation: $(D^2 + DD' - 6D'^2)z = y\cos x$, where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}$ **(12 Marks)**

46. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0,\ x = a,\ y = 0$ and $y = b$. The edges and $x = 0, x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at $100^0$C. **(30 Marks)**

47. Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method. **(15 Marks)**

48. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ canonical form. **(15 Marks)**

# 2007

49. (i) Form a partial differential equation by eliminating the function $f$ from:
$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$
(ii) Solve $2zx - px^2 - 2qxy + pq = 0$ **(6+6=12 Marks)**

50. Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. **(12 Marks)**

51. Solve $u_{xx} + u_{yy} = 0$ in $D$ where $D = \{(x, y) : 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions:

$u(x,0)=0,\ u(x,b)=0,\ 0\le x\le a$

$u(0,y)=g(y), u_x(a,y)=h(y),\ 0\le y\le b.$ **(30 Marks)**

52. Solve the equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions: $u(0,t)=0=u(l,t),$ for all $t$ and $u(x,0)=f(x)$ for all $x$ in $[0,\ l]$ **(30 Marks)**

# 2006

53. Solve : $px(z-2y^2)=(z-qy)(z-y^2-2x^3)$ **(12 Marks)**

54. Solve : $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial x\partial y^2}=2\sin(3x+2y)$ **(12 Marks)**

55. The deflection of vibrating string of length $l$, is governed by the partial differential equation $u_{tt}=C^2\ u_{xx}$. The ends of the string are fixed at and $x=0$ and $l$. The initial velocity is zero. The initial displacement is given by $u(x,0)=\begin{cases}\frac{x}{l}, & 0<x<\frac{l}{2}\\ \frac{1}{l}(l-x), & \frac{l}{2}<x<l.\end{cases}$

Find the deflection of the string at any instant of time. **(30 Marks)**

56. Find the surface passing through the parabolas $z=0,\ y^2=4ax$ and $z=1,\ y^2=-4ax$ and satisfying the equation $x\frac{\partial^2 z}{\partial x^2}+2\frac{\partial z}{\partial x}=0$ **(15 Marks)**

57. Solve the equation $p^2x+q^2y=z,\ p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ **(15 Marks)**

# 2005

58. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. **(12 Marks)**

59. Find the particular integral of $x(y-z)p+y(z-x)q=z(x-y)$ which represents a surface passing through $x=y=z$ **(12 Marks)**

60. The ends A and B of a rod 20cm long have the temperature at $30^0$C and $80^0$C until steady state prevails. The temperatures of ends are changed to $40^0$C and $60^0$C respectively. Find the temperature distribution in the rod at time $t$. **(30 Marks)**

61. Obtain the general solution of $(D-3D'-2)^2z=2e^{2x}\sin(y+3x)$ where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$ **(30 Marks)**

# 2004

62. Find the integral surface of the following partial differential equation : $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ **(12 Marks)**

63. Find the complete integral of the partial differential equation $(p^2+q^2)x = pz$ and deduce the solution which passes through the curve $x=0,\ z^2=4y$. **(12 Marks)**

64. Solve the partial differential equation : $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$ **(15 Marks)**

65. A uniform string of length $l$, held tightly between $x=0$ and $x=l$ with no initial displacement, is struck at $x=a,\ 0<a<l$, with velocity $v_0$. Find the displacement of the string at any time $t>0$ **(30 Marks)**

66. Using Charpit's method, find the complete solution of the partial differential equation $p^2x+q^2y=z$ **(15 Marks)**

# 2003

67. Find the general solution of $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y)$ **(12 Marks)**

68. Show that the differential equations of all cones which have their vertex at the origin are $px+qy=z$. Verify that $yz+zx+xy=0$ is a surface satisfying the above equation. **(12 Marks)**

69. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-3\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=xy+e^{x+2y}$ **(15 Marks)**

70. Solve the equation $p^2-q^2-2px-2qy+2xy=0$ using Charpit's method. Also find the singular solution of the equation, if it exists. **(15 Marks)**

71. Find the deflection $u(x,\ t)$ of a vibrating string, stretched between fixed points $(0,\ 0)$ and $(3l,\ 0)$, corresponding to zero initial velocity and following initial deflection:

$$f(x)=\begin{cases}\frac{hx}{l} & \text{when } 0\le x\le 1\\ \frac{h(3l-2x)}{l} & \text{when } l\le x\le 2l\\ \frac{h(x-3l)}{l} & \text{when } 2l\le x\le 3l\end{cases}$$

Where $h$ is a constant. **(15 Marks)**

# 2002

72. Find two complete integrals of the partial differential equation $x^2p^2+y^2q^2-4=0$ **(12 Marks)**

73. Find the solution of the equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ **(12 Marks)**

74. Frame the partial differential equation by eliminating the arbitrary constants $a$ and $b$ from $\log(az-1)=x+ay+b$ **(10 Marks)**

75. Find the characteristic strips of the equation $xp+yq-pq=0$ and then find the equation of the integral surface through the curve $z=\frac{x}{2},\ y=0$ **(20 Marks)**

76. Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ 0<x<l,\ t>0$

$$u(0,\ t)=u(l,\ t)=0$$
$$u(x,\ 0)=x(l-x),\quad 0\le x\le t.$$

**(30 Marks)**

# 2001

77. Find the complete integral partial differential equation $2p^2q^2+3x^2y^2=8x^2q^2(x^2+y^2)$ **(12 Marks)**

78. Find the general integral of the equation $\left\{my(x+y)-nz^2\right\}\frac{\partial z}{\partial x}-\left\{lx(x+y)-nz^2\right\}\frac{\partial z}{\partial y}=(lx-my)z$

**(12 Marks)**

79. Prove that for the equation $z+px+qy-1-pqx^2y^2=0$ the characteristic strips are given by

$x(t)=\dfrac{1}{B+Ce^{-t}},\ y(t)=\dfrac{1}{A+De^{-t}},\ z(t)=E-(AC+BD)e^{-t}$

$p(t)=A(B+Ce^{-t})^2,\ q(t)=B(A+De^{-t})^2$ where $A,B,C,D$ and $E$ are arbitrary constants. Hence find the values of these arbitrary constants if the integral surface passes through the line $z=0,\ x=y$

**(30 Marks)**

80. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2)=C_1y^2$ **(10 Marks)**

81. Solve the equation $x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=x^2y^4$ by reducing it to the equation with constant coefficients. **(20 Marks)**

# 2000

82. Solve : $pq=x^my^nz^{2l}$ **(12 Marks)**

83. Prove that if $x_1^3+x_2^3+x_3^3=1$ when $z=0$, the solution of the equation $(S-x_1)p_1+(S-x_2)p_2+(S-x_3)p_3=S-z$ can be given in the form

$S^3\left\{(x_1-z)^3+(x_2-z)^3+(x_3-z)^3\right\}^4=\left(x_1+x_2+x_3-3z\right)^3$ where $S=x_1+x_2+x_3+z$ and

$p_i=\dfrac{\partial z}{\partial x_i},\ i=1,2,3.$ **(12 Marks)**

84. Solve by Charpit's method the equation $p^2x(x-1)+2pqxy+q^2y(y-1)-2pxz-2qyz+z^2=0$

**(15 Marks)**

85. Solve : $(D^2-DD'-2D'^2)z=2x+3y+e^{3x+4y}$. **(15 Marks)**

86. A tightly stretched string with fixed end points $x=0,\ x=l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point $x$ of it a velocity $kx(l-x)$, obtain at time $t$ the displacement $y$ at a distance $x$ from the end $x=0$ **(30 Marks)**

# 1999

87. Verify that the differential equation $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable and find its primitive. **(20 Marks)**

88. Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$, $c$ is constant, orthogonally and which passes through the circle $x^2+y^2=1,\ z=1$ **(20 Marks)**

89. Find the characteristics of the equation $pq = z,$ and determine the integral surface which passes through the passes through the parabola $x = 0,\ y^2 = z.$ **(20 Marks)**

90. Use Charpit's method to find a complete integral to $p^2 + q^2 - 2px - 2qy + 1 = 0$ **(20 Marks)**

91. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-x}\cos y$ which $\to 0$ as $x \to \infty$ and has the value $\cos y$ when $x = 0$ **(20 Marks)**

92. One end of a string $(x = 0)$ is fixed, and the point $x = a$ is made to oscillate, so that at time $t$ the displacement is $g(t)$. Show that the displacement $u(x,\ t)$ of the point $x$ at time $t$ is given by

$u(x,\ t) = f(ct - x) - f(ct + x)$ where $f$ is a function satisfying the relation $f(t + 2a) = f(t) - g\left(\frac{t+a}{c}\right)$

**(20 Marks)**

# 1998

93. Find the differential equation of the set of all right circular cones whose axes coincide with the $z$-axis **(20 Marks)**

94. Form the differential equation by eliminating $a, b$ and $c$ from $z = a(x + y) + b(x - y) + abt + c$

**(20 Marks)**

95. Solve $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = xyz$ **(20 Marks)**

96. Find the integral surface of the linear partial the differential equation

$x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$ through the straight line $x + y = 0,\ z = 1$ **(20 Marks)**

97. Use Charpit's method to find a complete integral of $2x\left[\left(z\frac{\partial z}{\partial y}\right)^2 + 1\right] = z\frac{\partial z}{\partial x}$ **(20 Marks)**

98. Find a real function $V(x, y),$ which reduces to zero when $y = 0$ and satisfies the equation

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ **(20 Marks)**

99. Apply Jacobi's method to find a complete integral of the equation

$2x\frac{\partial z}{\partial x_1}x_1x_3 + 3\frac{\partial z}{\partial x_2}x_3^2 + x\left(\frac{\partial z}{\partial x_2}\right)^2 x\frac{\partial z}{\partial x_3} = 0$ **(20 Marks)**

# 1997

100. (i) Find the differential equation of all surfaces of revolution having $z$-axis as the axis of rotation.

(ii) Form the differential equation by eliminating $a$ and $b$ from $z = (x^2 + a)(y^2 + b)$

**(10+10=20 Marks)**

101. Find the equation of surfaces satisfying $4yzp + q + 2y = 0$ and passing through

$y^2 + z^2 = 1,\ x + z = 2$ **(20 Marks)**

102. Solve : $(y + z)p + (z + x)q = x + y$ **(20 Marks)**

103. Use Charpit's method to find complete integral of $z^2(p^2z^2 + q^2) = 1$ **(20 Marks)**

104. Solve : $(D_x{}^3 - D_y{}^3)z = x^3y^3$ **(20 Marks)**

105. Apply Jacobi's method to find complete integral of $p_1^{\ 3} + p_2^{\ 2} + p_3 = 1$. Here $p_1 = \frac{\partial z}{\partial x_1}$, $p_2 = \frac{\partial z}{\partial x_2}$, $p_3 = \frac{\partial z}{\partial x_3}$ and $z$ is a function of $x_1, x_2, x_3$. **(20 Marks)**

# 1996

106. (i) differential equation of all spheres of radius $\lambda$ having their center in $xy$- plane

(ii) Form differential equation by eliminating $f$ and $g$ from $z = f(x^2 - y) + g(x^2 + y)$

**(10+10=20 Marks)**

107. Solve : $z^2(p^2 + q^2 + 1) = C^2$ **(20 Marks)**

108. Find the integral surface of the equation $(x-y)y^2 p + (y-x)x^2 q = (x^2 + y^2)z$ passing through the curve $xz = a^3,\ y = 0$ **(20 Marks)**

109. Apply Charpit's method to find the complete integral of $z = px + qy + p^2 + q^2$ **(20 Marks)**

110. Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$ **(20 Marks)**

111. Find a surface passing through the lines $z = x = 0$ and $z - 1 = x - y = 0$ satisfying $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$ **(20 Marks)**

# 1995

112. In the context of a partial differential equation of the first order in there independent variables, define and illustrate the terms:

(i) The complete integral

(ii) The singular integral **(20 Marks)**

113. Find the general integral of $(y+z+w)\frac{\partial w}{\partial x} + (z+x+w)\frac{\partial w}{\partial y} + (x+y+w)\frac{\partial w}{\partial z} = x+y+z$ **(20 Marks)**

114. Obtain the differential equation of the surfaces which are the envelopes of a one-parameter family of planes. **(20 Marks)**

115. Explain in detail the Charpit's method of solving the nonlinear partial differential equation $f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$ **(20 Marks)**

116. Solve $\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2}\frac{\partial z}{\partial x_3} = z^3 x_1 x_2 x_3$ **(20 Marks)**

117. Solve $(D_x^{\ 3} - 7D_x D_y^{\ 2} - 6D_y^{\ 3})z = \sin(x+2y) + e^{3x+y}$ **(20 Marks)**

# 1994

118. Find the differential equation of the family of all cones with vertex at $(2,-3,1)$ **(20 Marks)**

119. Find the integral surface of $x^2 p + y^2 q + z^2 = 0,\ p \equiv \frac{\partial z}{\partial x},\ q \equiv \frac{\partial z}{\partial y}$ which passes through the hyperbola $xy = x + y,\ z = 1$ **(20 Marks)**

120. Obtain a Complete Solution of $pq = x^m y^n z^{2l}$ **(20 Marks)**

121. Use the Charpit's method to solve $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$. Interpret geometrically the complete solution and mention the singular solution. **(20 Marks)**

122. Solve $(D^2 + 3DD' + 2D'^2)z = x + y$, by expanding the particular integral in ascending powers of $D$, as well as in ascending powers of $D'$. **(20 Marks)**

123. Find a surface satisfying $(D^2 + DD')z = 0$ and touching the elliptic paraboloid $z = 4x^2 + y^2$ along its section by the plane $y = 2x + 1$. **(20 Marks)**

# 1993

124. Find the surface whose tangent planes cut off an intercept of constant length $R$ from the axis of $z$. **(20 Marks)**

125. Solve $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$ **(20 Marks)**

126. Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ through the circle $z = 1,\ x^2 + y^2 = 1$ **(20 Marks)**

127. Using Charpit's method find the complete integral of $2xz - px^2 - 2qxy + pq = 0$ **(20 Marks)**

128. Solve $r - s + 2q - z = x^2y^2$ **(20 Marks)**

129. Find the general solution of $x^2r - y^2t + xp - yq = \log x$ **(20 Marks)**

# 1992

130. Solve:
$$(2x^2 - y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + 2z^2 - yz - zx - 2xy)$$
**(20 Marks)**

131. Find the complete integral of $(y - x)(qy - px) = (p - q)^2$ **(20 Marks)**

132. Use Charpit's method to solve $px + qy = z\sqrt{1 + pq}$ **(20 Marks)**

133. Find the surface passing through the parabolas $z = 0,\ y^2 = 4ax;\ z = 1,\ y^2 = -4ax$ and satisfying the differential equation $xr + 2p = 0$ **(20 Marks)**

134. Solve : $r + s - 6t = y\cos x$ **(20 Marks)**

135. Solve: $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y) + e^y$ **(20 Marks)**