

Vector Analysis

Previous year Questions from
2017 to 1992

Ramanasri Institute

2017

- For what values of the constant a , b and c the vector $\vec{V} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of the vector with these values. [10 Marks]
- The position vector of a moving point at time t is $\vec{r} = \sin t \hat{i} + \cos 2t \hat{j} + (t^2 + 2t) \hat{k}$. Find the components of acceleration \vec{a} in the direction parallel to the velocity vector \vec{v} and perpendicular to the plane of \vec{r} and \vec{v} at time $t = 0$. [10 Marks]
- Find the curvature vector and its magnitude at any point $\vec{r} = (\theta)$ of the curve $\vec{r} = (a \cos \theta, a \sin \theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$. [16 Marks]
- Evaluate the integral $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 3xy^2\hat{i} + (yx^2 - y^3)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \leq 4$, $-3 \leq x \leq 3$ using divergence theorem. [9 Marks]
- Using Green theorem evaluate the $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise where $\vec{F}(\vec{r}) = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$ and $d\vec{r} = x\hat{i} + dy\hat{j}$ and the curve C is the boundary off the region $R = \{(x, y) | 1 \leq y \leq 2 - x^2\}$. [8 Marks]

2016

- Prove that the vector $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle find the length of the medians of the triangle [10 marks]
- Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$ [10 marks]
- Prove that $\oint_C f d\vec{r} = \iint_S d\vec{S} \times \nabla f$ [10 marks]
- For the cardioid $r = a(1 + \cos \theta)$ show that the square of the radius of curvature at any point (r, θ) is proportion to r . Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. [15 marks]

2015

- Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ [10 Marks]
- A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. [12 Marks]
- Evaluate $\int_C e^{-x} (\sin y dx + \cos y dy)$, where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $(\pi, \frac{\pi}{2})$, $(0, \frac{\pi}{2})$ [12 Marks]

2014

13. Find the curvature vector at any point of the curve $\vec{r}(t) = t \cos t \hat{i} + t \sin t \hat{j}$, $0 \leq t \leq 2\pi$. Give its magnitude also. [10 Marks]
14. Evaluate by Stokes' theorem $\int_{\Gamma} (y dx + z dy + x dz)$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$ starting from $(2a, 0, 0)$ and then going below the z -plane. [20 Marks]

2013

15. Show the curve $\vec{x}(t) = t \hat{i} + \left(\frac{1+t}{t}\right) \hat{j} + \left(\frac{1-t^2}{t}\right) \hat{k}$ lies in a plane. [10 Marks]
16. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n , r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator [10 Marks]
17. A curve in space is defined by the vector equation $\vec{r} = t^2 \hat{i} + 2t \hat{j} - t^3 \hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. [10 Marks]
18. By using Divergence Theorem of Gauss, evaluate the surface integral $\iint (a^2 x^2 + b^2 y^2 + c^2 z^2)^{\frac{1}{2}} dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. [15 Marks]
19. Use Stokes' theorem to evaluate the line integral $\int_C (-y^3 dx + x^3 dy - z^3 dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. [15 Marks]

2012

20. If $\vec{A} = x^2 y z \hat{i} - 2x z^3 \hat{j} + x z^2 \hat{k}$, $\vec{B} = 2z \hat{i} + y \hat{j} - x^2 \hat{k}$ find the value of $\frac{\partial^2}{\partial x \partial y} (\vec{A} + \vec{B})$ at $(1, 0, -2)$. [12 Marks]
21. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \frac{2}{3} t^3$. Show that the curvature and torsion are equal for this curve. [20 Marks]
22. Verify Green's theorem in the plane for $\oint_C [xy + y^2 dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [20 Marks]
23. If $\vec{F} = y \hat{i} + (x - 2xz) \hat{j} - xy \hat{k}$, evaluate $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n} ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy -plane. [20 Marks]

2011

24. For two vectors \vec{a} and \vec{b} give respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin 5t\hat{i} - \cos t\hat{j}$ determine: (i) $\frac{d}{dt}(\vec{a} \cdot \vec{b})$ and (ii) $\frac{d}{dt}(\vec{a} \times \vec{b})$ [10 Marks]
25. If u and v are two scalar fields and \vec{f} is a vector field, such that $u\vec{f} = \text{grad}v$, find the value of $\vec{f} \text{curl} \vec{f}$ [10 Marks]
26. Examine whether the vectors $\nabla u, \nabla v$ and ∇w are coplanar, where u, v and w are the scalar functions defined by :
- $$u = x + y + z,$$
- $$v = x^2 + y^2 + z^2$$
- and $w = yz + zx + xy$ [15 Marks]
27. If $\vec{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ calculate the double integral $\iint (\nabla \times \vec{u}) d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$ [15 Marks]
28. If \vec{r} be the position vector of a point, find the value(s) of n for which the vector $r^n \vec{r}$ is (i) irrotational, (ii) solenoidal [15 Marks]
29. Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 1$. [15 Marks]

2010

30. Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at the point (2,1) in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis. [12 Marks]
31. Show that the vector field defined by the vector function $\vec{v} = xyz(yz\hat{i} + xy\hat{j} + xy\hat{k})$ is conservative. [12 Marks]
32. Prove that $\text{div}(f\vec{V}) = f(\text{div}\vec{V}) + (\text{grad}.f)\vec{V}$ where f is a scalar function. [20 Marks]
33. Use the divergence theorem to evaluate $\iint_S \vec{V} \cdot n dA$ where $\vec{V} = x^2z\hat{i} + y\hat{j} - xz^2\hat{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. [20 Marks]
34. Verify Green's theorem for $e^{-x} \sin y dx + e^{-x} \cos y dy$ by the path of integration being the boundary of the square whose vertices are $(0,0), \left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ [20 Marks]

2009

35. Show that $\text{div}(\text{grad}r^n) = n(n+1)r^{n-2}$ where $r = \sqrt{x^2 + y^2 + z^2}$. [12 Marks]

36. Find the directional derivative of (i) $4xz^3 - 3x^2y^2z^2$ (ii) $-x^2yz + 4xz^2$ at $(2, -1, 2)$ along z-axis (i) at $(2, -1, 2)$ along z-axis (ii) $-x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. [6+6=12 Marks]
37. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ under the field of force of given by $\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$. [20 Marks]
38. Using divergence theorem, evaluate $\iiint_s \vec{A} \cdot d\vec{S}$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ [20 Marks]
39. Find the value of $\iiint_s (\vec{\nabla} \times \vec{f}) \cdot d\vec{s}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when $\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$ [20 Marks]

2008

40. Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. [12 Marks]
41. Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for \vec{F} and the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$. [12 Marks]
42. Prove that $\nabla^2 f(x) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r) = 0$. [15 Marks]
43. Show that for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$ the curvature and torsion are same at every point. [15 Marks]
44. Evaluate $\int_c \vec{A} \cdot d\vec{r}$ along the curve $x^2 + y^2 = 1, z = 1$ from $(0, 1, 1)$ to $(1, 0, 1)$ if $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$. [15 Marks]
45. Evaluate $\iint_s \vec{F} \cdot \hat{n} \, ds$ where $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}, \iint_s \vec{F} \cdot \hat{n} \, ds$ and S is the surface of the cylinder bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ [15 Marks]

2007

46. If \vec{r} denotes the position vector of a point and if \hat{r} be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$ determined $\text{grad}(r_{-1})$ in terms of \hat{r} and r . [12 Marks]
47. Find the curvature and torsion at any point of the curve $x = a \cos 2t, y = a \sin 2t, z = 2a \sin t$. [12 Marks]
48. For any constant vector, show that the vector \vec{a} represented by $\text{curl}(\vec{a} \times \vec{r})$ is always parallel to the vector \vec{a}, \vec{r} being the position vector of a point (x, y, z) measured from the origin. [15 Marks]

49. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find the value(s) of n in order that $r^n \vec{r}$ may be (i) solenoidal (ii) irrotational [15 Marks]
50. Determine $\int_C (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2, x+y=2a$ that starts from the point $(2a,0,0)$ goes at first below the z-plane. [15 Marks]

2006

51. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^2$ at the point $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to z-axis. [12 Marks]
52. If $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}, \vec{C} = 4\vec{i} - 3\vec{j} - 7\vec{k}$ determine a vector \vec{R} satisfying the vector equation $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ & $\vec{R} \cdot \vec{A} = 0$ [15 Marks]
53. Prove that $r^n \vec{r}$ is an irrotational vector for any value of n but is solenoidal only if $n+3=0$ [15 Marks]
54. If the unit tangent vector \vec{t} and binormal \vec{b} make angles ϕ and ψ respectively with a constant unit vector \vec{a} prove that $\frac{\sin \theta}{\sin \phi} \cdot \frac{d\theta}{d\phi} = -\frac{k}{\tau}$. [15 Marks]
55. Verify Stokes' theorem for the function $\vec{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z=0$ and bounded by the lines $x=0, y=0, x=a$ and $y=a, a>0$. [15 Marks]

2005

56. Show that the volume of the tetrahedron ABCD is $\frac{1}{6}(\vec{AB} \times \vec{AC}) \cdot \vec{AD}$ Hence find the volume of the tetrahedron with vertices $(2,2,2), (2,0,0), (0,2,0)$ and $(0,0,2)$ [12 Marks]
57. Prove that the curl of a vector field is independent of the choice of coordinates [12 Marks]
58. The parametric equation of a circular helix is $r = a \cos u \hat{i} + a \sin u \hat{j} + cu \hat{k}$ where c is a constant and u is a parameter. Find the unit tangent vector \hat{t} at the point u and the arc length measured from $u=0$ Also find $\frac{d\hat{t}}{ds}$ where s is the arc length. [15 Marks]
59. Show that $\text{curl} \left(k \times \text{grad} \frac{1}{r} \right) + \text{grad} \left(k \cdot \text{grad} \frac{1}{r} \right) = 0$ where r is the distance from the origin and k is the unit vector in the direction OZ [15 Marks]
60. Find the curvature and the torsion of the space curve [15 Marks]
61. Evaluate by Gauss divergence theorem, where S is the surface of the cylinder bounded by and [15 Marks]

2004

62. Show that if \vec{A} and \vec{B} are irrotational, then $\vec{A} \times \vec{B}$ is solenoidal. [12 Marks]
63. Show that the Frenet-Serret formulae can be written in the form $\frac{d\vec{T}}{ds} = \vec{\omega} \times \vec{T}, \frac{d\vec{N}}{ds} = \vec{\omega} \times \vec{N}$ & $\frac{d\vec{B}}{ds} = \vec{\omega} \times \vec{B}$, where $\vec{\omega} = \tau \vec{T} + k \vec{B}$. [12 Marks]

64. Prove the identity $\nabla(\bar{A} \cdot \bar{B}) = (\bar{B} \cdot \nabla)\bar{A} + (\bar{A} \cdot \nabla)\bar{B} + \bar{B} \times (\nabla \times \bar{A}) + \bar{A} \times (\nabla \times \bar{B})$ [15 Marks]
65. Derive the identity $\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \iint_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n} dS$ where V is the volume bounded by the closed surface S. [15 Marks]
66. Verify Stokes' theorem for $\hat{f} = (2x - y)\hat{i} - yz^2\hat{j}z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. [15 Marks]

2003

67. Show that if a' , b' and c' are the reciprocals of the non-coplanar vectors a , b and c , then any vector r may be expressed as $r = (r \cdot a')a + (r \cdot b')b + (r \cdot c')c$. [12 Marks]
68. Prove that the divergence of a vector field is invariant w, r, to co-ordinate transformations. [12 Marks]
69. Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions. [15 Marks]
70. Prove the identity $\nabla A^2 = 2(A \cdot \nabla)A + 2A \times (\nabla \times A)$ where $\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$ [15 Marks]
71. Find the radii of curvature and torsion at a point of intersection of the surface $x^2 - y^2 = c^2, y = x \tanh\left(\frac{z}{c}\right)$. [15 Marks]
72. Evaluate $\iint_S \text{curl } A \cdot ds$ where S is the open surface $x^2 + y^2 - 4x + 4z = 0, z \geq 0$ and $A = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$. [15 Marks]

2002

73. Let \bar{R} be the unit vector along the vector $\bar{r}(t)$ Show that $\bar{R} \times \frac{d\bar{R}}{dt} = \frac{\bar{r}}{r^2} \times \frac{d\bar{r}}{dt}$ where $r = |\bar{r}|$ [12 Marks]
74. Find the curvature k for the space curve $x = a \cos \theta, y = a \sin \theta, z = a \theta \tan \alpha$ [15 Marks]
75. Show that $(\text{curl } \bar{v}) = \text{grad}(\text{div } \bar{v}) - \nabla^2 \bar{v}$. [15 Marks]
76. Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it. Show that $\iint_S (f \text{grad } f) \cdot \hat{n} ds = \iiint_V [|\text{grad } f|^2 + f \nabla^2 f] dv$ Hence $\iint_S (f \text{grad } f) \cdot \hat{n} ds$ or otherwise evaluate for $f = 2x + y + 2z$ over $s = x^2 + y^2 + z^2 = 4$ [15 Marks]
77. Find the values of constants a, b and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at $(1, -1, 1)$ is in the direction parallel to y -axis and has magnitude 6. [15 Marks]

2001

78. Find the length of the arc of the twisted curve $r = (3t, 3t^2, 2t^3)$ from the point $t = 0$ to the point $t = 1$ Find also the unit tangent t , unit normal n and the unit binormal b at $t = 1$. [12 Marks]

79. Show that $\text{curl} \frac{a \times r}{r^3} = -\frac{a}{r^3} + \frac{3r}{r^5}(a \cdot r)$ where a is constant vector. [12 Marks]
80. Find the directional derivative of $f = x^2 y z^3$ along $x = e^{-t}$, $y = 1 + 2 \sin t$, $z = t - \cos t$ at $t = 0$ [15 Marks]
81. Show that the vector field defined by $F = 2xyz^3 i + x^2 z^3 j + 3x^2 yz^2 k$ is irrotational. Find also the scalar u such that $F = \text{grad } u$ [15 Marks]
82. Verify Gauss' divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. [15 Marks]

2000

83. In what direction from the point $(-1, 1, 1)$ is the directional derivative $f = x^2 y z^3$ of a maximum? Compute its magnitude. [12 Marks]
84. Show that the covariant derivatives of the fundamental metric tensors $g_{ij}, g^{ij}, \delta^i_j$ Vanish (ii) Show that simultaneity is relative in special relativity theory. [6+6=12 Marks]
85. Show that
 (i) $(A+B) \cdot (B+C) \times (C+A) = 2A \cdot B \times C$
 (ii) $\nabla \times (A \times B) = (B \cdot \nabla) A - B(\nabla \cdot A) - (A \cdot \nabla) B + A(\nabla \cdot B)$ [7+8=15 Marks]
86. Evaluate $\iint_S F \cdot N ds$ where $F = 2xyi + yz^2 j + xzk$ and S is the surface of the parallelepiped bounded by $x = 0, y = 0, z = 0, x = 2, y = 1$ and $z = 3$ [15 Marks]
87. If g_{ij} and γ_{ij} are two metric tensors defined at a point and $\left\{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \right\}$ and $\left[\begin{smallmatrix} l \\ ij \end{smallmatrix} \right]$ are the corresponding Christoffel symbols of the second kind, then prove that $\left\{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \right\} - \left[\begin{smallmatrix} l \\ ij \end{smallmatrix} \right]$ is a mixed tensor of the type A^l_{ij} [15 Marks]
88. Establish the formula $E = mc^2$ the symbols have their usual meaning. [15 Marks]

1999

89. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of A, B, C prove that $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$ is vector perpendicular to the plane ABC [20 Marks]
90. If $\bar{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\nabla \times \bar{f}$. [20 Marks]
91. Evaluate $\int_c (e^{-x} \sin y dx + e^{-x} \cos y dy)$ (by Green's theorem), where C is the rectangle whose vertices are $(0, 0), (\pi, 0), (\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$ [20 Marks]

1998

92. If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point $P(x, y, z)$ then the values of $\text{grad}(r_1 r_2)$ and $\text{curl}(r_1 \times r_2)$. [20 Marks]

93. Show that $(a \times b) \times c = a \times (a \times b)$ if either $b = 0$ (or any other vector is 0) or c is collinear with a or b is orthogonal to a and c (both) [20 Marks]
94. Prove that $\left\{ \begin{matrix} i \\ ik \end{matrix} \right\} = \frac{\partial}{\partial x_k} (\log \sqrt{g})$. [20 Marks]

1997

95. Prove that if \vec{A}, \vec{B} and \vec{C} are three given non-coplanar vectors \vec{F} then any vector can be put in the form $F = \alpha \vec{B} \times \vec{C} + \beta \vec{C} \times \vec{A} + \gamma \vec{A} \times \vec{B}$ for given determine α, β, γ . [20 Marks]
96. Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ [20 Marks]
97. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij}T_{ij}$ of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti-symmetric part of T_{ij} . [20 Marks]

1996

98. State and prove 'Quotient law' of tensors [20 Marks]
99. If $x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ show that
 (i) $\vec{r} \times \text{grad}f(r) = 0$
 (ii) $\text{div}(r^n \vec{r}) = (n+3)r^n$ [20 Marks]
100. Verify Gauss's divergence theorem for $\vec{F} = xyx\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z, x + y + z = 1$. [20 Marks]

1995

101. Consider a physical entity that is specified by twenty-seven numbers A_{ijk} in given coordinate system. In the transition to another coordinate system of this kind. Let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk} - A_{jik}$ are the components of a tensor B_{jk} of third order. Is the component of tensor? Give reasons for your answer [20 Marks]
102. Let the region V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S . If ϕ is harmonic in V , show that $\int_S \frac{\partial \phi}{\partial n} ds = 0$ [20 Marks]
103. In the vector field $u(x)$ let there exist a surface $curl v$ on which $v = 0$. Show that, at an arbitrary point of this surface $curl v$ is tangential to the surface or vanishes. [20 Marks]

1994

104. Show that $r^n \vec{r}$ is an irrotational vector for any value of n , but is solenoidal only if $n = -3$. [20 Marks]
105. If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$ evaluate $\iint_S (\Delta \times \vec{F}) \cdot \vec{n} ds$ Where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane. [20 Marks]
106. Prove that $\left\{ \begin{matrix} i \\ ik \end{matrix} \right\} = \frac{\partial}{\partial x} (\log \sqrt{g})$. [20 Marks]

1993

107. Prove that the angular velocity or rotation at any point is equal to one half of the curl of the velocity vector V . [20 Marks]
108. Evaluate $\iint_S \Delta \times \vec{F} \cdot \vec{n} ds$ where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ [20 Marks]
109. Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one [20 Marks]

1992

110. If $\vec{F}(x, y, z) = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k}$ then calculate $\int_C \vec{F} \cdot d\vec{x}$ where C consists of
 (i) The line segment from $(0, 0, 0)$ to $(1, 1, 1)$ (ii) the three line segments AB, BC and CD where A, B, C and D are respectively the points $(0, 0, 0), (1, 0, 0), (1, 1, 0)$ and $(1, 1, 1)$ (iii) the curve $\vec{x} + u\vec{i} + u^2\vec{j} + u^2\vec{k}, u$ from 0 to 1. [20 Marks]
111. If \vec{a} and \vec{b} are constant vectors, show that
 (i) $\text{div} \{ \vec{x} \times (\vec{a} \times \vec{x}) \} = -2\vec{x} \cdot \vec{a}$
 (ii) $\text{div} \{ \vec{x} \times (\vec{a} \times \vec{x}) \times (\vec{b} \times \vec{x}) \} = 2\vec{a} \cdot (\vec{b} \times \vec{x}) - 2\vec{b} \cdot (\vec{a} \times \vec{x})$ [20 Marks]
112. Obtain the formula $\text{div} \vec{A} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left\{ \left(\frac{g}{g_{ij}} \right)^{1/2} A^i \right\}$ where A^i are physical components of \vec{A} and use it to derive expression of $\text{div} \vec{A}$ in cylindrical polar coordinates [20 Marks]