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UPSC MATHS OPTIONAL COACHING

Analytical Geometry

Previous year Questions from 2020 to 1992

2021-22

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2020

1. Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9$ **[10 Marks]**

2. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4, z = 2$ **[15 Marks]**

3. If the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then find the equations of the other two generators. **[15 Marks]**

4. Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$ **[15 Marks]**

2019

5. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+2}{2}$ intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. **[10 Marks]**

6. The plane $x + 2y + 3z = 12$ cuts the axes of coordinates in. A, B, C Find the equations of the circle circumscribing the triangle ABC **[10 Marks]**

7. Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has vertex at $(2, 4, 1)$ in a rectangular hyperbola. **[10 Marks]**

8. Prove that, in general, three normal can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$ but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then two of the three normals coincide. **[15 Marks]**

9. Find the length of the normal chord through a point P of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and prove that if it is equal to $4PG_3$ where G_3 is the point where the normal chord through P meets xy plane, then P lies on the cone $\frac{x^2}{a^6}(2c^2 - a^2) + \frac{y^2}{b^6}(2c^2 - b^2) + \frac{z^2}{c^4} = 0$ **[15 Marks]**

2018

10. Find the projection of the straight line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ on the plane $x + y + 2z = 6$ **[10 Marks]**

11. Find the shortest distance between the lines $a_1x + b_1y + c_1z + d_1 = 0,\ a_2x + b_2y + c_2z + d_2 = 0$ and the z-axis. **[12 Marks]**

12. Find the equations to the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point $(1,1,1)$ **[13 Marks]**

13. Find the equation of the sphere in xyz -plane passing through the points $(0,0,0),(0,1,-1),(-1,2,0)$ and $(1,2,3)$ **[12 Marks]**.

14. Find the equation of the cone with $(0,0,1)$ as the vertex and $2x^2-y^2=4, z=0$ as the guiding curve. **[13 Marks]**

15. Find the equation of the plane parallel to $3x-y+3z=8$ and passing through the point $(1,1,1)$ **[12 Marks]**

2017

16. Find the equation of the tangent at the point $(1,1,1)$ to the Conicoid $3x^2-y^2=2z$. **[10 Marks]**

17. Find the shortest distance between the skew the lines: $\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ **[10 Marks]**

18. A plane through a fixed point (a,b,c) and cuts the axes at the points A,B,C respectively. Find the locus of the center of the sphere which passes through the origin O and A,B,C **[15 Marks]**

19. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ find the point of contact. **[10 Marks]**

20. Find the locus of the points of intersection of three mutually perpendicular tangent planes to $ax^2+by^2+cz^2=1.$ **[10 Marks]**

21. Reduce the following equation to the standard form and hence determine the nature of the Conicoid: $x^2+y^2+z^2-yz-zx-xy-3x-6y-9z+21=0$. **[15 Marks]**

2016

22. Find the equation of the sphere which passes though the circle $x^2+y^2=4; z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3. **[10 marks]**

23. Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{4}=z-3$ and $y-mx=z=0$ for what value of will the two lines intersect? **[10 marks]**

24. Find the surface generated by a line which intersects the line $y=a=z, x+3z=a=y+z$ and parallel to the plane $x+y=0$. **[10 marks]**

25. Show that the cone $3yz-2zx-2xy=0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1}=\frac{y}{1}=\frac{z}{z}$ is a generator belonging to one such set, Find the other two. **[10 marks]**

26. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the Conicoid $ax^2+by^2+cz^2=1$. **[15 marks]**

2015

27. Find what positive value of a, the plane $ax-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and hence find the point of contact. **[10 Marks]**

28. If $6x = 3y = 2z$ represents one of the mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators. **[13 Marks]**

29. Obtain the equation of the plane passing through the points $(2,3,1)$ and $(4,-5,3)$ parallel to x- axis **[6 Marks]**

30. Verify if the lines: $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar. If yes, find the equation of the plane in which they lie. **[7 Marks]**

31. Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight-line touch. **[13 Marks]**

2014

32. Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines **[10 Marks]**

33. Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4,$ the tangent planes at which are parallel to the plane $2x - y + 2z = 1$ **[10 Marks]**

34. Prove that equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ **[10 Marks]**

35. Show that the lines drawn from the origin parallel to the normals to the central Conicoid $ax^2 + by^2 + cz^2 = 1,$ at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2$$ **[15 Marks]**

36. Find the equations of the two generating lines through any point $(a\cos\theta, b\sin\theta, 0)$ of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ of the hyperboloid by the plane $z = 0$ **[15 Marks]**

2013

37. Find the equation of the plane which passes through the points $(0,\ 1,\ 1)$ and $(2,\ 0,\ -1)$ and is parallel to the line joining the points $(-1,\ 1,\ -2),\ (3,\ -2,\ 4)$. Find also the distance between the line and the plane. **[10 Marks]**

38. A sphere S has points $(0,\ 1,\ 0)\ (3,\ -5,\ 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. **[10 Marks]**

39. Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$ **[15 Marks]**

40. A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0,\ z = 0$ and passes through a fixed point $(0,\ 0,\ c)$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that vertex lies one the fixed circle $x^2 + y^2 + 2ax + 2by = 0, 2ax + 2by + cz = 0$ **[15 Marks]**

41. A variable generator meets two generators of the system through the extremities B and B^1 of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2c^2 = 1$ in P and P^1 Prove that $BP.\ P^1B^1 = a^2 + c^2$ **[20 Marks]**

2012

42. Prove that two of the straight lines represented by the equation $x^3 + bx^2y + cxy^2 + y^3 = 0$ will be at right angles, if $b + c = -2$ **[12 Marks]**

43. A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C respectively. Prove that circle ABC lies on the cone $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$ **[20 Marks]**

44. Show that locus of a point from which three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z^2 = 0$ is $x^2 + y^2 + 4z = 1$ **[20 Marks]**

2011

45. Find the equation of the straight line through the point $(3,\ 1,\ 2)$ to intersect the straight line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$ **[10 Marks]**

46. Show that the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point $(1,\ 2,\ -2)$ and the passes through the point $(-1,\ 0,\ 0)$ is $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ **[10 Marks]**

47. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3,\ 1,\ -1)$ **[20 Marks]**

48. Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that lines joining to P, Q and R to origin are mutually perpendicular. Prove that plane PQR touches a fixed sphere **[20 Marks]**

49. Show that the cone $yz + xz + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area **[20 Marks]**

50. Show that generators through any one of the ends of an equi-conjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are inclined to each other at an angle of 60^0 if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. **[20 Marks]**

2010

51. Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle **[12 Marks]**

52. Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact **[20 Marks]**

53. Show that every sphere through the circle $x^2 + y^2 - 2ax + r^2 = 0,\ z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2, y = 0$ **[20 Marks]**

54. Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid $\frac{x^2}{4} + y^2 - z^2 = 49$ passing through $(10,\ 5,\ 1)$ and $(14,\ 2,\ -2)$. **[20 Marks]**

2009

55. A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0$ to meet two fixed lines $y=mx,\ z=c$ and $y=-mx,\ z=-c$. Find the locus of the line **[12 Marks]**

56. Find the equation of the sphere having its center on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-12x-3y+4z+8=0,\ 3x+4y-5z+3=0$ **[12 Marks]**

57. Prove that the normals from the point $(\alpha,\ \beta,\ \gamma)$ to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$ **[20 Marks]**

2008

58. The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$; find the equation of the plane in its new position **[12 Marks]**

59. Find the equations (in symmetric form) of the tangent line to the sphere $x^2+y^2+z^2+5x-7y+2z-8=0$, $3x-2y+4z+3=0$ at the point $(-3,\ 5,\ 4)$. **[12 Marks]**

60. A sphere S has points $(0,\ 1,\ 0),(3,\ -5,\ 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle **[20 Marks]**

61. Show that the enveloping cylinders of the ellipsoid $a^2x^2+b^2y^2+c^2z^2=1$ with generators perpendicular to z- axis meet the plane $z=0$ in parabolas. **[20 Marks]**

2007

62. Find the equation of the sphere inscribed in the tetrahedron whose faces are $x=0,\ y=0,\ z=0$ and $2x+3y+6z=6$ **[12 Marks]**

63. Find the locus of the point which moves so that its distance from the plane $x+y-z=1$ is twice its distance from the line $x=-y=z$ **[12 Marks]**

64. Show that the spheres $x^2+y^2+z^2-x+z-2=0$ and $3x^2+3y^2-8x-10y+8z+14=0$ cut orthogonally. Find the center and radius of their common circle **[15 Marks]**

65. A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{4}$ and $\frac{x-11}{3}=\frac{y-5}{-1}=\frac{z}{1}$. Find the coordinate of the points of intersection and the length intercepted on it **[15 Marks]**

66. Show that the plane $2x-y+2z=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines **[15 Marks]**

67. Show that the feet of the normals from the point $P(\alpha,\ \beta,\ \gamma),\beta\neq 0$ on the paraboloid $x^2+y^2=4z$ lie on the sphere $2\beta(x^2+y^2+z^2)-(\alpha^2+\beta^2)y-2\beta(2+\gamma)z=0$ **[15 Marks]**

2006

68. A pair of tangents to the conic $ax^2+by^2=1$ intercepts a constant distance $2k$ on the y- axis. Prove that the locus of their point of intersection is the conic $ax^2(ax^2+by^2-1)=bk^2(ax^2-1)^2$ **[12 Marks]**

69. Show that the length of the shortest distance between the line $z=x\tan\alpha, y=0$ and any tangent to the ellipse $x^2\sin^2\alpha+y^2=a^2, z=0$ is constant **[12 Marks]**

70. If PSP^1 and QSQ^1 are the two perpendicular focal chords of a conic $\frac{1}{r}=1+e\cos\theta$, Prove that $\frac{1}{SP.SP^1}+\frac{1}{SQ.SQ^1}$ is constant **[15 Marks]**

71. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point $(1,\ -2,\ 1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$ **[15 Marks]**

72. Show that the plane $ax+by+cz=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines, if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ **[15 Marks]**

73. If the plane $lx+my+nz=p$ passes through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ prove that $a^2l^2+b^2m^2+c^2n^2=3p^2$ **[15 Marks]**

2005

74. If normals at the points of an ellipse whose eccentric angles are α,β,γ and δ in a point then show that $\sin(\beta+\gamma)+\sin(\gamma+\alpha)+\sin(\alpha+\beta)=0$ **[12 Marks]**

75. A square $ABCD$ having each diagonal AC and BD of length $2a$ is folded along the diagonal AC so that the planes DAC and BAC are at right angle. Find the shortest distance between AB and DC **[12 Marks]**

76. A plane is drawn through the line $x+y=1, z=0$ to make an angle $\sin^{-1}\left(\frac{1}{3}\right)$ with plane $x+y+z=5$. Show that two such planes can be drawn. Find their equations and the angle between them. **[15 Marks]**

77. Show that the locus of the centers of sphere of a co-axial system is a straight line. **[15 Marks]**

78. Obtain the equation of a right circular cylinder on the circle through the points $(a,\ 0,\ 0),(0,\ \mathrm{b},\ 0),(0,\ 0,\ \mathrm{c})$ as the guiding curve. **[15 Marks]**

79. Reduce the following equation to canonical form and determine which surface is represented by it:
$x^2-7y^2+2z^2-10yz-8zx-10xy+6x+12y-6z+2=0$ **[15 Marks]**

2004

80. Prove that the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2=4ax$ is $(x+a)y^2+x^3=0$. **[12 Marks]**

81. Find the equations of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0,$ which are parallel to the plane $2x+y-z=4$ **[12 Marks]**

82. Find the locus of the middle points of the chords of the rectangular hyperbola $x^2-y^2=a^2$ which touch the parabola $y^2=4ax$ **[15 Marks]**

83. Prove that the locus of a line which meets the lines $y=\pm mx,\ z=\pm c$ and the circle $x^2+y^2=a^2,\ z=0$ is
$c^2m^2(cy-mzx)^2+c^2(yz-cmx)^2=a^2m^2(z-c^2)^2$ **[15 Marks]**

84. Prove that the lines of intersection of pairs of tangent planes to $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone $a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0$ **[15 Marks]**

85. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point $(\alpha,\ \beta,\ \gamma)$. Prove that the perpendiculars to them through the origin generate the cone $(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$ **[15 Marks]**

2003

86. A variable plane remains at a constant distance unity from the point $(1,\ 0,\ 0)$ and cuts the coordinate axes at $A, B,$ and $C,$ find the locus of the center of the sphere passing through the origin and the point and the point A, B and C. **[12 Marks]**

87. Find the equation of the two straight lines through the point $(1,\ 1,\ 1)$ that intersect the line $x-4=4(y-4)=2(z-1)$ at an angle of 60^0 **[12 Marks]**

88. Find the volume of the tetrahedron formed by the four planes $lx+my+nz=p, lx+my=0, my+nz=0$ and $nz+lx=0$ **[15 Marks]**

89. A sphere of constant radius r passes through the origin O and cuts the co-ordinate axes at A, B and C. Find the locus of the foot of the perpendicular from O to the plane ABC. **[15 Marks]**

90. Find the equations of the lines of intersection of the plane $x+7y-5z=0$ and the cone $3xy+14zx-30xy=0$ **[15 Marks]**

91. Find the equations of the lines of shortest distance between the lines: $y+z=1,\ x=0$ and $x+z=1,\ y=0$ as the intersection of two planes **[15 Marks]**

2002

92. Show that the equation $9x^2-16y^2-18x-32y-151=0$ represents a hyperbola. Obtain its eccentricity and foci. **[12 Marks]**

93. Find the co-ordinates of the center of the sphere inscribed in the tetrahedron formed by the plane $x=0,\ y=0,\ z=0$ and $x+y+z=a$ **[12 Marks]**

94. Tangents are drawn from any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the circle $x^2+y^2=r^2$. Show that the chords of contact are tangents to the ellipse $a^2x^2+b^2y^2=r^2$. **[15 Marks]**

95. Consider a rectangular parallelepiped with edges a, b and c. Obtain the shortest distance between one of its diagonals and an edge which does not intersect this diagonal **[15 Marks]**

96. Show that the feed of the six normals drawn from any point $(\alpha,\ \beta,\ \gamma)$ to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the cone $\frac{a^2(b^2-c^2)\alpha}{x}+\frac{b^2(c^2-a^2)\beta}{y}+\frac{c^2(a^2-b^2)\gamma}{z}=0$ **[15 Marks]**

97. A variable plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ is parallel to the plane meets the co-ordinate axes of A, B and C. Show that the circle ABC lies on the conic $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$ **[15 Marks]**

2001

98. Show that the equation $x^2-5xy+y^2+8x-20y+15=0$ represents a hyperbola. Find the coordinates of its center and the length of its real semi-axes. **[12 Marks]**

99. Find the shortest distance between the axis of z and the lines $ax+by+cz+d=0,\ a^1x+b^1y+c^1z+d^1=0$ **[12 Marks]**

100. Find the equation of the circle circumscribing the triangle formed by the points $(a,\ 0,\ 0),(0,\ \mathrm{b},\ 0),(0,\ 0,\ \mathrm{c})$. Obtain also the coordinates of the center of the circle. **[15 Marks]**

101. Find the locus of equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **[15 Marks]**

102. Prove that $5x^2+5y^2+8z^2+8yz+8zx-2xy+12x-12y+6=0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$ **[15 Marks]**

103. If TP,TQ and T^1P^1,T^1Q^1 all lie one a conic. **[15 Marks]**

2000

104. Find the equations to the planes bisecting the angles between the planes $2x-y-2z=0$ and $3x+4y+1=0$ and specify the one which bisects the acute angle. **[12 Marks]**

105. Find the equation to the common conjugate diameters of the conics $x^2+4xy+6y^2=1$ and $2x^2+6xy+9y^2=1$ **[12 Marks]**

106. Reduce the equation $x^2+y^2+z^2-2xy-2yz+2zx+x-y-2z+6=0$ into canonical form and determine the nature of the quadric **[15 Marks]**

107. Find the equation of the sphere through the circle $x^2+y^2+z^2=4, x+2y-z=2$ and the point $(1,-1,1)$ **[15 Marks]**

108. A variable straight line always intersects the lines $x=c, y=0; y=c, z=0; z=c, x=0$. Find the equations to its locus **[15 Marks]**

109. Show that the locus of mid-points of chords of the cone $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ drawn parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is the plane $(al+hm+gn)x+(hl+bm+fn)y+(gl+fm+cn)z=0$ **[15 Marks]**

1999

110. If P and D are ends of a pair of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ show that the tangents at P and D meet on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ **[20 Marks]**

111. Find the equation of the cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are perpendicular to the plane $x-y-3z=5$. **[20 Marks]**

112. Calculate the curvature and torsion at the point u of the curve given by the parametric equations $x=a(3u-u^3),\ y=3au^2,\ z=a(3u+u^2)$ **[20 Marks]**

1998

113. Find the locus of the pole of a chord of the conic $\frac{l}{r}=1+e\cos\theta$ which subtends a constant angle 2α at the focus **[20 Marks]**

114. Show that the plane $ax+by+cz+d=0$ divides the join of $P_1\equiv(x_1,\ y_1,\ z_1), P_2\equiv(x_2,\ y_2,\ z_2)$ in the ratio $-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$. Hence show that the planes $U\equiv ax+by+cz+d=0=a^1x+b^1y+c^1z+d^1\equiv V$, $U+\lambda V=0$ and $U-\lambda V=0$ divide any transversal harmonically **[20 Marks]**

115. Prove that a curve $x(s)$ is a generalized helix if and only if it satisfies the identity $x^{ii}.x^{iii}\times x^{iv}=0$ **[20 Marks]**

116. Find the smallest sphere (i.e. the sphere of smallest radius) which touches the lines $\frac{x-5}{2}=\frac{y-2}{-1}=\frac{z-5}{-1}$ and $\frac{x+4}{-3}=\frac{y+5}{-6}=\frac{z-4}{4}$ **[20 Marks]**

117. Find the co-ordinates the point of intersection of the generators $\frac{x}{a}-\frac{y}{b}-2\lambda=0=\frac{x}{a}-\frac{y}{b}-\frac{z}{\lambda}$ and $\frac{x}{a}+\frac{y}{b}-2\mu=0=\frac{x}{a}-\frac{y}{b}-\frac{z}{\mu}$ of the surface $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$. Hence show that the locus of the points of intersection of perpendicular generators curves of intersection of the surface with the plane $2z+(a^2-b^2)=0$ **[20 Marks]**

118. Let $P\equiv(x',\ y',\ z')$ lie on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. If the length of the normal chord through P is equal to $4PG$, where G is the intersection of the normal with the z- plane, then show that P lies on the cone $\frac{x^2}{a^6}(ac^2-a^2)+\frac{y^2}{b^6}(ac^2-b^2)+\frac{z^2}{c^4}=0$ **[20 Marks]**

1997

119. Let P be a pint on an ellipse with its center at the point C. Let CD and CP be two conjugate diameters. If the normal at P cuts CD in F, show that $CD.PF$ is a constant and the locus of F is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=\left[\frac{a^2-b^2}{x^2+y^2}\right]^2$ where $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ equation of the given ellipse **[20 Marks]**

120. A circle passing through the focus of conic section whose latus rectum is $2l$ meets the conic in four points whose distances from the focus are $\gamma_1,\gamma_2,\gamma_3$ and γ_4 Prove that $\frac{1}{\gamma_1}+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}+\frac{1}{\gamma_4}=\frac{2}{l}$ **[20 Marks]**

121. Determine the curvature of the circular helix $\vec{r}(t)=(a\cos t)\hat{i}+a(\sin t)\hat{j}+(bt)\hat{k}$ and an equation of the normal plane at the point $\left(0,a,\frac{\pi b}{2}\right)$. **[20 Marks]**

122. Find the reflection of the plane $x+y+z-1=0$ in plane $3x+4z+1=0$ **[20 Marks]**

123. Show that the pint of intersection of three mutually perpendicular tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lies on the sphere $x^2+y^2+z^2=a^2+b^2+c^2$ **[20 Marks]**

124. Find the equation of the spheres which pass through the circle $x^2+y^2+z^2-4x-y+3z+12=0,$ $2x+3y-7z=10$ and touch the plane $x-2y+2z=1$ **[20 Marks]**

1996

125. Find the equation of the common tangent to the parabolas $y^2=4ax$ and $x^2=4by$ **[20 Marks]**

126. If the normal at any point 't_1' of a rectangular hyperbola $xy=c^2$ meets the curve again at the point 't_2', prove that $t_1^{\ 3}t_2=-1$. **[20 Marks]**

127. A variable plane is at a constant distance p from the origin and meets the axes in A,B and C. Through A,B,C the planes are drawn parallel to the coordinate planes. Show that the locus of their point of intersection is given by $x^{-2}+y^{-2}+z^{-2}=p^{-2}$ **[20 Marks]**

128. Find the equation of the sphere which passes through the points $(1,0,0),(0,1,0),(0,0,1)$ and has the smallest possible radius. **[20 Marks]**

129. The generators through a point P on the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meet the principal elliptic section in two points such that the eccentric angle of one is double that of the other. Show that P lies on the curve $x=\frac{a(1-3t^2)}{1+t^2}, y=\frac{bt(3-t^2)}{1+t^2}, z=ct$ **[20 Marks]**

130. A curve is drawn on a right circular cone, semi-vertical angle α, so as to cut all the generators at the same angle β. Show that its projection on a plane at right angles to the axis is an equiangular spiral. Find expressions for its curvature and torsion. **[20 Marks]**

1995

131. Two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the circle $x^2+y^2=r^2$ at P and Q. Show that the locus of middle point of PQ is $a^2\left\{(x^2+y^2)^2-r^2x^2\right\}+b^2\left\{(x^2+y^2)^2-r^2y^2\right\}=0$ **[20 Marks]**

132. If the normal at one of the extremities of latus rectum of the conic $\frac{1}{r}=1+e\cos\theta$, meets the curve again at Q, show that $SQ=\frac{l(1+3e^2+e^4)}{(1+e^2-e^4)}$, where S is the focus of the conic. **[20 Marks]**

133. Through a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes in A,B,C. Prove that the area of the triangle ABC is $\frac{r^2}{2x'y'z'}$ where r is the measure of OP. **[20 Marks]**

134. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the area of the common circle is $\frac{\pi r_1^2 r_2^2}{r_1^2+r_2^2}$ **[20 Marks]**

135. Show that a plane through one member of the λ- system and one member of μ- system is tangent plane to the hyperboloid at the point of intersection of the two generators. **[20 Marks]**

136. Prove that the parallels through the origin to the binormals of the helix $x=a\cos\theta,\ y=a\sin\theta,\ z=k\theta$ lie upon the right cone $a^2(x^2+y^2)=k^2z^2$.

1994

137. If 2φ be the angle between the tangents from $P(x_1,y_1)$ to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, prove that $\lambda_1\cos^2\varphi+\lambda_2\sin^2\varphi=0$ where λ_1,λ_2 are the parameters of two con-foci to the ellipse through P **[20 Marks]**

138. If the normals at the points $\alpha,\beta,\gamma,\delta$ on the conic $\frac{l}{r}=1+e\cos\theta$ meet at (ρ,ϕ), prove that $\alpha+\beta+\gamma+\delta-2\phi=$ odd multiple of π radians. **[20 Marks]**

139. A variable plane is at a constant distance p from the origin O and meets the axes in A,B and C. Show that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$ **[20 Marks]**

140. Find the equations to the generators of hyperboloid, through any point of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, z=0$ **[20 Marks]**

141. Planes are drawn through a fixed point (α,β,γ) so that their sections of the paraboloid $ax^2+by^2=2z$ are rectangular hyperbolas. Prove that they touch the cone $\frac{(x-\alpha^2)}{b}+\frac{(y-\beta^2)}{a}+\frac{(z-\gamma^2)}{a+b}=0$. **[20 Marks]**

142. Find $f(\theta)$ so that the curve $x=a\cos\theta,\ y=a\sin\theta,\ z=f(\theta)$ determines a plane curve. **[20 Marks]**

1993

143. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of lines, prove that the area of the triangle formed by their bisectors and axis of x is $\sqrt{\frac{(a-b)^2+4h^2}{2h}},\ \frac{ca-g^2}{ab-h^2}$ **[20 Marks]**

144. Find the equation of the director circle of the conic $\frac{l}{r}=1+e\cos\theta$ and also obtain the asymptotes of the above conic. **[20 Marks]**

145. A line makes angles $\alpha,\ \beta,\ \gamma,\delta$ with the diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$ **[20 Marks]**

146. Prove that the center of the spheres which touch the lines $y=mx,\ z=c;\ y=-mx,\ z=-c$ lie upon the Conicoid $mxy+cz(1+m^2)=0$ **[20 Marks]**

147. Find the locus of the point of intersection of perpendicular generators of a hyperboloid of one sheet. **[20 Marks]**

148. A curve is drawn on a parabolic cylinder so as to cut all the generators at the same angle. Find its curvature and torsion. **[20 Marks]**

1992

149. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two intersecting lines, show that the square of the distance of the point of intersection of the straight lines from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}(ab-h^2\neq 0)$ **[20 Marks]**

150. Discuss the nature of the conic $16x^2-24xy+9y^2-104x-172y+144=0$ in detail **[20 Marks]**

151. A straight line, always parallel to the plane of yz, passed through the curves $x^2+y^2=a^2,\ z=0$ and $x^4=ax,\ y=0$ prove that the equation of the surface generated is $x^4y^2=(x^2-az)^2(a^2-x^2)$ **[20 Marks]**

152. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point $(\alpha,\ \beta,\ \gamma)$. Prove that the perpendicular them from the origin generate the cone $(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$ **[20 Marks]**

153. Show that the locus of the foot of the perpendicular from the center to the plane through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $a^2x^2+b^2y^2+c^2z^2=3(x^2+y^2+z^2)$ **[20 Marks]**

154. Define an osculating plane and derive its equation in vector form. If the tangent and binormal at a point P of the curves make angles $\theta,\ \phi$ respectively with the fixed direction, show that $\left(\frac{\sin\theta}{\sin\phi}\right)\left(\frac{d\theta}{d\phi}\right)=-\frac{k}{\tau}$ where k and τ are respectively curvature and torsion of the curve at P.