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Linear Algebra

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

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2020

1. If A is a skew-symmetric matrix and $I+A$ be a non-singular matrix, then show that $(I-A)(I+A)^{-1}$ is orthogonal. **[8 Marks]**

2. By applying elementary row operations on the matrix $A=\begin{bmatrix}-1 & 2 & -1 & 0\\ 2 & 4 & 4 & 2\\ 0 & 0 & 1 & 5\\ 1 & 6 & 3 & 2\end{bmatrix}$, reduce it to a row-reduced echelon matrix. Hence find the rank of A **[8 Marks]**

3. Let $T: R^3 \to R^3$ be defined by $T(x,y,z)=(2x,-3y,x+y)$ and $B_1=\{(-1,2,0),(0,1,-1),(3,1,2)\}$ be a basis of R^3. Find the matrix representation of T relative to the basis B_1. **[10 Marks]**

4. When is a matrix A said to be similar to another matrix B?
Prove that
(i) If A is similar to B, then B is similar to A.
(ii) two similar matrices have the same eigenvalues.

Further, by choosing appropriately the matrices A and B, show that the converse of (ii) above may not be true. **[15 Marks]**

5. (i) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ hence find its inverse. Also, express $A^5-4A^4-7A^3+11A^2-A-10,I$ as a Linear polynomial in A
(ii) Express the vector $(1,2,5)$ as a linear combination of the vectors $(1,1,1),(2,1,2)$ and $(3,2,3)$ if possible. Justify your answer **[9+6=15 Marks]**

2019

6. Let $\mathrm{T}: \mathrm{R}^3 \to \mathrm{R}^3$ be a linear operator on R^3 defined by $\mathrm{T}(x,\ y,\ z)=(2y+z,\ x-4y,3x)$ Find the matrix of T in the basis $\{(1,\ 1,\ 1),(1,\ 1,\ 0),(1,\ 0,\ 0)\}$. **[8 Marks]**

7. The Eigen values of a real symmetric matrix A are $-1,\ 1$ and -2. The corresponding eigenvectors are $\frac{1}{\sqrt{2}}(-1\ 1\ 0)^{\mathrm{T}}$, $(0\ 0\ 1)^{\mathrm{T}}$ and $\frac{1}{\sqrt{2}}(-1\ -1\ 0)^{\mathrm{T}}$ respectively. Find the matrix A^4. **[8 Marks]**

8. Consider the singular matrix

$$\mathrm{A}=\begin{bmatrix}-1 & 3 & -1 & 1\\ -3 & 5 & 1 & -1\\ 10 & -10 & -10 & 14\\ 4 & -4 & -4 & 8\end{bmatrix}$$

Given that one Eigen value of A is 4 and one eigenvector that does not correspond to this Eigen value 4 is $(1\ 1\ 0\ 0)^{\mathrm{T}}$. Find all the Eigen values of A other than 4 and hence also find the real numbers $p,\ q,\ r$ that satisfy the matrix equation $\mathrm{A}^4+p\mathrm{A}^3+q\mathrm{A}^2+r\mathrm{A}=0$. **[15 Marks]**

9. (a) Consider the vectors $x_1 = (1, 2, 1, -1)$, $x_2 = (2, 4, 1, 1)$, $x_3 = (-1, -2, 0, -2)$ and $x_4 = (3, 6, 2, 0)$ in R^4. Justify that the linear span of the set $\{x_1, x_2, x_3, x_4\}$ is a subspace of R^4 defines as $\{(\xi_1, \xi_2, \xi_3, \xi_4) \in R^4 : 2\xi_1 - \xi_2 = 0,\ 2\xi_1 - 3\xi_3 - \xi_4 = 0\}$
Can this subspace be written as $\{(\alpha, 2\alpha, \beta, 2\alpha - 3\beta) : \alpha, \beta \in R\}$? What is the dimension of this subspace? **[15 Marks]**

10. (a) Using elementary row operations, reduce the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix}$$

To reduced echelon form and find the inverse of A and hence solve the system of linear equations $Ax = b$, where $X = (x, y, z, u)^T$ and $b = (2, 1, 0, 4)^T$. **[15 Marks]**

2018

11. Given that $\text{Adj}A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $\det A = 2$ Find the matrix A. **[8 Marks]**

12. Prove that the Eigen values of a Hermitian matrix are all real. **[8 Marks]**

13. Show that the matrices $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & 0 \end{bmatrix}$ are congruent. **[10 Marks]**

14. Show that the vectors $\alpha_1 = (1,0,-1)$,$\alpha_2 = (1,2,1), \alpha_3 = (0,-3,2)$ form a basis for R^3 Express each of the standard basis vectors as a linear combination of $\alpha_1, \alpha_2 \alpha_3$ **[10 Marks]**

15. Let $T : V_2(R) \to V_2(R)$ be a linear transformation defined by $T(a,b) = (a, a+b)$ Find the matrix of, T taking $\{e_1, e_2\}$ as a basis for the domain and $\{(1,1),(1,-1)\}$ as a basis for the range. **[10 Marks]**

16. If $(n+1)$ vectors $\alpha_1, \alpha_2, ..., \alpha_n, \alpha$ form a linearly dependent set, then show that the vector α is a linear combination of $\alpha_1, \alpha_2, ..., \alpha_n$; provided $\alpha_1, \alpha_2, ..., \alpha_n$ form a linearly independent set. **[10 Marks]**

2017

17. Let A be a square matrix of order 3 such that each of its diagonal elements is 'a' and each of its off-diagonal elements is 1. If $B = bA$ is orthogonal determine the values of a and b. **[8 Marks]**

18. Let V be the vector space of all 2×2 matrices over the field R. Show that W is not a subspace of V, where
(i) W contains all 2×2 matrices with zero determine.
(ii) W consists of all 2×2 matrices A such that $A^2 = A$ **[8 Marks]**

19. Using the Mean Value theorem, show that

(i) f(x) is constant in $[a,b]$ if $f'(x)=0$ in $[a,b]$

(ii) f(x) is a decreasing function in (a,b) if $f'(x)$ exists and is <0 everywhere in (a,b) **[8 Marks]**

20. State the Cayley –Hamilton theorem. Verify this theorem for the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$ Hence find A^{-1} **[10 Marks]**

21. Reduce the following matrix to a row-reduce echelon form and hence find its rank:

$$A=\begin{bmatrix}-1 & 2 & -1 & 0\\ 2 & 4 & 4 & 2\\ 0 & 0 & 1 & 5\\ 1 & 6 & 3 & 2\end{bmatrix}$$ **[10 Marks]**

22. Given that the set $\{u,v,w\}$ is linearly independent examine the sets

(i) $\{u+v, v+w, w+u\}$ F

(ii) $\{u+v, u-v, u-2v+2w\}$

for linear independence **[10 Marks]**

23. Find the Eigen values and the corresponding eigenvectors for the matrix $A=\begin{bmatrix}0 & -2\\ 1 & 3\end{bmatrix}$ Examine whether the matrix A is diagonalizable. Obtain a matrix D (if it is diagonalizable) such that $D=p^{-1}AP$. **[10 Marks]**

2016

24. Let $T:\mathbb{R}^3\to\mathbb{R}^4$ be given by $T(x,y,z)=(2x-y, 2x+z, x+2z, x+y+z)$

Find the matrix of T with respect to standard basis of $\mathbb{R}^3$ and $\mathbb{R}^4$ (i.e., $(1,0,0),(0,1,0)$ etc.). Examine if T is a liner map. **[8 Marks]**

25. For the matrix $A=\begin{bmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{bmatrix}$, obtain the eight value and get the value of $A^4+3A^3-9A^2$ **[8 Marks]**

26. Let T be a linear map such that $T:V3\to V2$ defined by $T(e_1)=2f_1-f_2, T(e_2)=f_1+2f_2, T(e_3)=0f_1+0f_2$ where e_1,e_2,e_3 and f_1,f_2 are standard basis in V_3 and V_2 Find the matrix of T relative to theses basis. Further take two other basis $B_1[(1,1,0)(1,0,1)(0,1,1)]$, and $B_2[(1,1)(1,-1)]$ obtain the matrix T_1 relative to B_1 and B_2 . **[10 Marks]**

27. For the matrix $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ Find two non-singular matrices P and Q such that $PAQ=I$

Hence find A^{-1} . **[10 Marks]**

28. Examine whether the real quadratic form $4x^2-y^2+2z^2+2xy-2yz-4xz$ is a positive definite of not. Reduce it to its diagonal form and determine its signature. **[10 Marks]**

2015

29. (a) Find an upper triangular matrix A such that $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$ **[8 Marks]**

30. Let G be the liner operator on $\mathbb{R}^3$ defined by

$$G(x,y,z) = (2y+z, x-4y, 3x)$$

Find the matrix representation of G relative to the basis

$$S = \{(1,1,1),(1,1,0)(1,0,0)\}$$ **[8 Marks]**

31. Suppose U and W are distinct four-dimensional subspaces of a vector space V, where $\dim V = 6.$ Find the dimensional of $U \cap W$ **[10 Marks]**

32. Find the condition on a, b and c so that the following system in unknowns x, y and z has a solution:

$$x + 2y - 3z = a$$
$$2x + 6y - 11z = b$$
$$x - 2y + 7z = c$$

[10 Marks]

33. Find the minimal polynomial of the matrix $A = \begin{pmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ **[10 Marks]**

34. Find a 3×3 orthogonal matrix whose first two rows are $\left[\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right]$ and $\left[0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$ **[10 Marks]**

2014

35. Show that $u_1 = (1,-1,0), u_2 = (1,1,0)$ and $u_3 = (0,1,1)$ form a basis for $\mathbb{R}^3$ .Express $(5,3,4)$ in terms of u_1, u_2 and u_3 **[8 Marks]**

36. For the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Prove that $A^n = A^{n-2} + A^2 - I, n \geq 3$ **[8 Marks]**

37. Let $B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ find all Eigen values and corresponding Eigen vectors of B viewed as a matrix over:

(i)The real field R

(ii)The complex field C. **[10 Marks]**

38. Examine whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Find all Eigen values. Then obtain matrix P such that $P^{-1}AP$ is a diagonal matrix. **[10 Marks]**

39. Show that the mapping $T : V_2(\bar{R}) \to V_3(\bar{R})$ defined as $T(a,b) = (a+b, a-b, b)$ is a linear transformation .Find the range, rank and nullity of T **[10 Marks]**

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40. Consider the line mapping $F:\mathbb{R}^2 \to \mathbb{R}^2$ given as $F(x,y)=(3x+4y,2x-5y)$ with usual basis. Find the matrix associated with the linear transformation relative to the basis $S=\{u_1,u_2\}$ where $u_1=(1,2),u_2=(2,3)$ **[10 Marks]**

2013

41. Find the dimension and a basis of the solution space W of the system $x+2y+2z-s+3t=0, x+2y+3z+s+t=0, 3x+6y+8z+s+5t=0$ **[8 Marks]**

42. Find the characteristic equation of the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$ and hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ **[8 Marks]**

43. Let V be the vector space of 2×2 matrices over $\mathbb{R}$ and let $M=\begin{bmatrix}1&-1\\-2&2\end{bmatrix}$ let $V\to V$ be the linear map defined by $F(A)=MA$. Find a basis and the dimension of

(i) The kernel of W of F

(ii) The image U of F **[10 Marks]**

44. Locate the stationary point of the function $x^4+y^4-2x^2+4xy-2y^2$ and determine their nature. **[10 Marks]**

45. Find an orthogonal transformation of co-ordinates which diagonalizes the quadratic form $q(x,y)=2x^2-4xy+5y^2$ **[10 Marks]**

46. Discuss the consistency and the solutions of the equation

$$x+ay+az=1, ax+y+2az=-4, ax-ay+4z=2$$

for different values of a. **[10 Marks]**

47. Let F be a subfield of complex number and T a function from $F^3\to F^3$ defined by $T(x_1,x_2,x_3)=(x_1+x_2+3x_3, 2x_1-x_2, -3x_1+x_2-x_3)$. What are the condition on (a,b,c) such that (a,b,c) be in the null space of T? Find the nullity of T. **[10 Marks]**

48. Let $H=\begin{bmatrix}1&i&2+i\\-i&2&1-i\\2-i&1+i&2\end{bmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $P^tH\overline{P}$ is diagonal and also find signature. **[10 Marks]**

2012

49. Let $V=\mathbb{R}^3$ and $\alpha_1=(1,1,2), \alpha_2=(0,1,3), \alpha_3=(2,4,5)$ and $\alpha_4=(-1,0,-1)$ be the element of V. Find a basis for the intersection of the subspace spanned by $\{\alpha_1,\alpha_2\}$ and $\{\alpha_3,\alpha_4\}$. **[8 Marks]**

50. Show that the set of all function which satisfy the differential equation $\dfrac{d^2f}{dx^2}+3\dfrac{df}{dx}=0$ is a vector space. **[8 Marks]**

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51. Let $f:\mathbb{R}\to\mathbb{R}^3$ be a linear transformation defined by $f(a,b,c)=(a,a+b,0)$ Find the matrices A and B respectively of the liner transformation f with respect to the standard basis (e_1,e_2,e_3) and the basis (e'_1,e'_2,e'_3) where $e'_1=(1,1,0),e'_2=(0,1,1),e'_3=(1,1,1)$. Also show that there exists an invertible matrix P such that $B=P^{-1}AP$ **[10 Marks]**

52. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$ and find its inverse. Also express

$A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A. **[10 Marks]**

53. Show that there are three real values of λ for which the equation: $(a-\lambda)x+by+cz=0,bx+(c-\lambda)y+az=0,cx+ay+(b-\lambda)z=0$ are simultaneously true and that the product of these values of λ is $D=\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}$ **[10 Marks]**

54. Find the matrix representation of linear transformation T on $V_3(\mathbb{R})$ defined as $T(a,b,c)=(2b+c,a-4b,3a)$ corresponding to the basis $B=\{(1,1,1),(1,1,0),(1,0,0)\}$. **[10 Marks]**

2011

55. Let V be the vector space of 2×2 matrices over the field of real number R. Let $W=\{A\in V\,|\,\text{trace A}=0\}$ show that W is a subspace of V. Find a basis of W and dimension of W

[10 Marks]

56. Find the linear transformation from R^3 into R^3 which has its range the subspace spanned by $(1,0,-1),(1,2,2)$ **[10 Marks]**

57. Let

$V=\{(x,y,z,u)\in R^4 : y+z+u=0\}$,

$W=\{(x,y,z,u)\in R^4 : x+y=0,z=2u\}$

be two subspaces of R^4 Find bases for $V,W,V+W$ and $V\cap W$ **[10 Marks]**

58. Find the characteristic polynomial of the matrix $A=\begin{pmatrix}3&1&1\\2&4&2\\-1&-1&1\end{pmatrix}$ and hence Compute A^{10}

[10 Marks]

59. Let $A=\begin{pmatrix}1&-3&3\\0&-5&6\\0&-3&4\end{pmatrix}$ find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. **[10 Marks]**

60. Find an orthogonal transformation to reduce the quadratic form $5x^2+2y^2+4xy$ to a canonical form. **[10 Marks]**

2010

61. Show that the set $P[t]=\{at^2+bt+c / a, b, c\in\mathbb{R}\}$ forms a vector space over the field $\mathbb{R}$ find a basis for this vector space what is the dimension of this vector space? **[8 Marks]**

62. Determine whether the quadratic form $q=x^2+y^2+2xz+4yz+3z^2$ is positive definite. **[8 Marks]**

63. Show that the vectors $\alpha_1=(1,0,-1),\alpha_2=(1,2,1),\alpha_3=(0,-3,2)$ form a basis for R^3 Find the components of $(1,0,0)\, w.r.t.$ the basis $\{\alpha_1,\alpha_2,\alpha_3\}$ **[10 Marks]**

64. Find the characteristic polynomial of $\begin{pmatrix}0&0&1\\1&0&2\\0&1&3\end{pmatrix}$ Verify Cayley-Hamilton theorem for this matrix and hence find its inverse. **[10 Marks]**

65. Let $A=\begin{pmatrix}5&-6&-6\\-1&4&2\\3&-6&-4\end{pmatrix}$. find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix **[12 Marks]**

66. Find the rank of the matrix $\begin{pmatrix}1&2&1&1&2\\2&4&3&4&7\\-1&-2&2&5&3\\3&6&2&1&3\\4&8&6&8&9\end{pmatrix}$ **[8 Marks]**

2009

67. Let V be the vector space of polynomials over R Let U and W be the subspaces generated by $\{t^3+4t^2-t+3, t^3+5t^2+5, 3t^3+10t^2-5t+5\}$

and

$\{t^3+4t^2+6, t^3+2t^2-t+5, 2t^3+2t^2-3t+9\}$

Respectively Find

(i) dim $(U+W)$

(ii) dim $(U\cap W)$ **[10 Marks]**

68. Find a linear map $T: R^3\to R^4$ whose image is generated $(1,2,0,-4)$ by and $(2,0,-1,-3)$ **[10 Marks]**

69. Let T be the linear operator on R^3 defined by $T(x,y,z)=(2x,4x-y,2x+3y-z)$.

(i) Show that T is invertible.

(ii) Find a formula for T^{-1} **[10 Marks]**

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70. Find the rank of the matrix: $A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$ **[10 Marks]**

71. Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ is A similar to a diagonal matrix? If so, find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. **[10 Marks]**

72. Find an orthogonal transformation of coordinates to reduce the quadratic form $q(x, y) = 2x^2 + 2xy + 2y^2$ to a canonical form **[10 Marks]**

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