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Mechanics & Fluid Dynamics

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

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2020

1. Find the condition on a,b,c (real numbers) such that the dynamical system with equations $\dot{p} = aq - q^2, \dot{q} = bp + cq$ is Hamiltonian. Compute also the Hamiltonian of the system. **[8 Marks]**
2. In a fluid flow, the velocity vector is given by $\vec{V} = 2x\vec{i} + 3y\vec{j} - 5z\vec{k}$. Determine the equation of the streamline passing through a point $A = (4,8,1)$. **[10 Marks]**
3. In a two-dimensional fluid flow, the velocity components are given by $u = x - \alpha y$ and $v = -\alpha x - y$, where α is constant. Show that the velocity potential exists for this flow and determine the appropriate velocity potential. Also, determine the corresponding stream function that would represent the flow. **[15 Marks]**
4. A particle is attracted to a center by a force which varies inversely as the cube of its distance from the center. Identify the generalized coordinates and write down the Lagrangian of the system. Derive then the equations of motion and solve then for the orbits. Discuss how the nature of orbits depends on the parameters of the system. **[20 Marks]**

2019

5. Consider the flow field given by $\psi = \mathrm{a}(\mathrm{x}^2 - \mathrm{y}^2)$, `a' being a constant. Show that the flow is irrotational. Determine the velocity potential for this flow and show that the streamlines and equi-velocity potential curves are orthogonal. **[8 Marks]**
6.
7. For a dynamical system $\mathrm{T} = \frac{1}{2}\left\{(1+2\mathrm{k})\dot{\theta}^2 + 2\dot{\theta}\dot{\varphi} + \dot{\varphi}^2\right\}, \quad \mathrm{V} = \frac{\mathrm{n}^2}{2}\left\{(1+\mathrm{k})\theta^2 + \varphi^2\right\}$, where θ, φ are coordinates and n, k are positive constants write down the Lagrange's equations of motion and deduce that $(\ddot{\theta} - \ddot{\varphi}) + \mathrm{n}^2\left(\frac{1+\mathrm{k}}{\mathrm{k}}\right)(\theta - \varphi) = 0$. Further show that if $\theta = \varphi, \dot{\theta} = \dot{\varphi}$ at $\mathrm{t} = 0$, then $\theta = \varphi$ for all t. **[15 Marks]**
8. Consider a mass-spring system consisting of a mass m and a linear spring of stiffness k hanging from a fixed point. Find the equation of motion using the Hamiltonian method, assuming that the displacement x is measured from the unstretched position of the string. **[10 Marks]**
9. Consider that the region $0 \le \mathrm{z} \le \mathrm{h}$ between the planes $\mathrm{z} = 0$ and $\mathrm{z} = \mathrm{h}$ is filled with viscous incompressible fluid. The plane $\mathrm{z} = 0$ is held at rest and the plane $\mathrm{z} = \mathrm{h}$ moves with constant velocity $\mathrm{V}\hat{\mathrm{j}}$. When conditions are steady, assuming there is no slip between the fluid and either boundary, and neglecting body forces, show that the velocity profile between the plates is parabolic. Find the tangential stress at any point $\mathrm{P}(\mathrm{x},\ \mathrm{y},\ \mathrm{z})$ of the fluid and determine the drag per unit area on both the planes. **[15 Marks]**

2018

10. Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and υ be the velocity at a distance x from a fixed point at time t, then $\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left\{\rho(\upsilon^2 + k)\right\}$ **[10 Marks]**

11. For a particle having charge q and moving in an electromagnetic field, the potential energy is $U = q(\phi - \vec{\upsilon}.\vec{A})$ where ϕ and $\vec{A}$ are, respectively, known as the scalar and vector potentials. Derive expression for Hamiltonian for the particle in the electromagnetic field. **[8 Marks]**

12. In the case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u in a fixed direction, where is u is a variable. Show that the maximum value of the velocity at any point of the fluid is $2u$. Prove that the force necessary to hold the disc is, $2m\dot{u}$ where m is the mass of the liquid displaced by the disc. **[12 Marks]**

13. A particle of mass m is constrained to move on the inner surface of a cone of semi-angle α under the action of gravity. Write the equation of constraint and mention the generalized coordinates. Write down the equation of motion. **[10 Marks]**

14. Two sources, each of strength m, are placed at the points $(-a,0),(a,0)$ and a sink of strength 2m at the origin. Show that the streamlines are the curves $(x^2+y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where, λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)/(r_1 r_2 r_3)$, where r_1, r_2, r_3 are the distances of the point from the sources and the sink. **[10 Marks]**

2017

15. A uniform rectangular parallelepiped of mass M has edges of lengths $2a, 2b, 2c$.Find the moment of inertia of this rectangular parallelepiped about the line through its centre parallel to the edge of length 2a. **[10 Marks]**

16. Consider a mass m on the end of a spring of natural length l and spring constant k.Let y be the vertical coordinate of the mass as measured from the top of the spring. Assume that the mass can only move up and down in the vertical direction show that $L = \frac{1}{2}my'^2 - \frac{1}{2}k(y-l)^2 + mgy$

Also determine and solve the corresponding Euler-Lagrange equation of motion. **[12 Marks]**

17. Find the streamlines and path lines of the two dimensional velocity fields: $u = \frac{x}{1+t}, v = y, w = 0$

[8 Marks]

18. The velocity vector in the flow field is given by $\vec{q} = (az - by)\hat{i} + (bx - cz)j + (cy - ax)k$ where a, b, c are non-zero constants. Determine the equation of vortex lines. **[8 Marks]**

2016

19. Calculate the moment of inertia of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(i) Relative to the x - axis

(ii) Relative to the y -axis and

(iii) Relative to the origin. **[8 Marks]**

20. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta=\theta_0$ .At time $t=0$ one end is suddenly cooled to $\theta=0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$ **[20 Marks]**

21. Find the moment of inertial of a right solid cone of mass M , height h and radius of whose base is a , about its axis.

[14Marks]

22. A bead slides on a wire in the shape of a cycloid described by the equation $x=a(\theta-\sin\theta)$, $y=a(1+\cos\theta)$ where $0\le\theta\le2\pi$ and the friction between the bead and the wire is negligible Deduce Lagrange's equation of motion. **[10 Marks]**

23. A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being P. If the radius R of the sphere varies in such a way that $R=a+b\cos nt$, where $b>a$, then find the pressure at the surface of the sphere at any time. **[16 Marks]**

2015

24. Find the moment of inertia of a uniform mass M of square shape with each side a about its one of the diagonals. **[12 Marks]**

25. Suppose $\vec{v}=(x-4y)\hat{i}+(4x-y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow. **[12 Marks]**

26. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection $u(x,0)=\begin{cases}\frac{2}{l}x, & \text{if } 0<x<\frac{l}{2}\\ \frac{2}{l}(l-x), & \text{if } \frac{l}{2}\le x<l\end{cases}$ with initial velocity $u_t(x,0)=0$.

[16 Marks]

27. In a steady fluid flow, the velocity components are $u=2kx, v=2ky$ and. $w=-4kz$. Find the equation of streamline passing though $(1,0,1)$ **[12 Marks]**

28. Derive the Hamiltonian and equation of motion for a simple pendulum. **[10 Marks]**

2014

29. Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation $\frac{D\bar{\Omega}}{Dt} = (\bar{\Omega}\cdot\nabla)\vec{q} + v\nabla^2\bar{\Omega}$ where $\vec{q}$ the velocity vector with $\vec{\Omega} = \nabla\times\vec{q}$. **[8 Marks]**

30. Find the condition that $f(x, y, \lambda) = 0$ should be a possible system of streamlines for steady irrotational motion in two dimension where λ is a variable parameter **[8 Marks]**

31. Show that the moment of inertia of uniform rectangular mass M and sides $2a$ and $2b$ and $2b$ about a diagonal is $\frac{2Ma^2b^2}{3(a^2+b^2)}$ **[10 Marks]**

32. A uniform rod OA of length $2a$ is free to turn about its end O, revolves with uniform angular velocity ω about a vertical axis OZ through O and is inclined at a constant angle α to OZ .Show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$ **[10 Marks]**

33. A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move .Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ where a is the length of the plank. **[15 Marks]**

34. Prove that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible from for the bounding surface of a liquid and find the velocity components. **[15 Marks]**

2013

35. Drive the Hamiltonian and equation of motion for a simple pendulum. **[14 Marks]**

36. Find the values of a and b in the $2-D$ velocity field $\vec{v} = (3y^2 - ax^2)\,\hat{i} + bxy\,\hat{j}$ so that the flow become incompressible and irrotational. Find the stream function of the flow. **[12 Marks]**

2012

37. Drive the differential equation of motion for a spherical pendulum. **[13 Marks]**

38. Prove that the voracity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation $\frac{D\bar{\Omega}}{Dt} = (\bar{\Omega}\cdot\nabla)\vec{q} + v\nabla^2\bar{\Omega},$ where υ is kinematic viscosity. **[10 Marks]**

39. Show that $u = \frac{A(x^2 - y^2)}{(x^2 + y^2)^2}, v = \frac{2Axy}{(x^2 + y^2)^2}, \quad w = 0$ are components of a possible velocity vector for in viscid incompressible fluid flow. Determine the pressure associated with this velocity field. **[13 Marks]**

40. A weightless rod ABC of length 2a is movable about the end A witch is fixed and carries two particle of mass m each one attached to the mid-point B of the rod and the attached to the end C of the rod. If the rod is held in the horizontal position and released from rest and allowed to move, sho that the angular velocity of the rod when it is vertical is $\sqrt{\frac{6g}{5a}}$ **[13 Marks]**

2011

41. Find the Lagrangian for a simple pendulum and obtain the equation describing its motion. **[10 Marks]**

42. With Usual notations, show that ϕ and ψ for a uniform flow past a stationary cylinder are given by

$$\phi = U\cos\theta\left(r + \frac{a^2}{r}\right),$$

$$\psi = U\sin\theta\left(r - \frac{a^2}{r}\right)$$ **[10 Marks]**

43. A uniform string of length l is held fixed between the point $x = 0$ and $x = l$ the two points of trisection are pulled aside though a distance ε on opposite sides of the equilibrium position and is released from rest at time $t = 0$ Find the displacement of the string at any latter time $t > 0$ what is the displacement of the string at the midpoint? **[16 Marks]**

44. For a steady poiseuille flow through a tube of uniform circular cross-section show that

$$w(R) = \frac{1}{4}\left(\frac{p}{\mu}\right)(a^2 - R^2)$$ **[16 Marks]**

45. From a uniform sphere of radius a, a spherical sector of vertical angle 2α is removed. Find the moment of inertia of the remainder mass M about the axis of symmetry. **[14 Marks]**

46. Is $\vec{q} = \frac{k^2(x\hat{j} + y\hat{i})}{x^2 + y^2}$ a possible velocity vector of an incompressible fluid motion? If so, find the stream function and velocity potential of the motion. **[14 Marks]**

2010

47. Show that the sum of the moments of inertia of an elliptic area about any two tangents at right angles is always the same. **[10 Marks]**

48. A two-dimensional flow field is given by $\psi = xy$. show that-

(i) The flow is irrotational;

(ii) ψ And ϕ satisfy Laplace equation.

Symbols ψ and ϕ convey the usual meaning **[10 Marks]**

49. Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two-dimensional fluid. Further show that the streamlines at time t are the curves $(x-t)^2 - (y-t)^2 =$ constant **[13 Marks]**

50. A mass m_1 hanging at the end of a string draws a mass m_2 along the surface of a smooth table. If the mass on the table be doubled, the tension of the string is increased by one half show that $m_1 : m_2 = 2:1$ **[13 Marks]**

51. Show that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation $\frac{D\vec{\Omega}}{Dt} = \left(\vec{\Omega}.\nabla\right)\vec{q} + \upsilon\nabla^2\vec{\Omega}.$ where v is the Kinematic viscosity **[14 Marks]**

2009

52. Show that for an incompressible steady flow with constant viscosity the velocity components $u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1-\frac{y}{h}\right), v = 0, w = 0$ satisfy the equation of motion when the body force is neglected $h, U, \frac{dp}{dx}$ are constants and $p = p(x)$. **[13 Marks]**

53. A homogeneous sphere of radius a, rotating with angular velocity about horizontal diameter is gently placed on a table whose coefficient of friction is μ. Show that there a will be slipping at the point of contact for a time $\frac{2\omega a}{7\mu g}$ and that then the sphere will roll with angular velocity $\frac{2\omega}{7}$. **[14 Marks]**

54. A cannon of mass M resting on a rough horizontal plane of coefficient of friction μ is fired with such a charge that the relative velocity of the ball and cannon at the moment when it leaves the cannon is μ show that the cannon will recoil a distance $\left(\frac{mu}{M+m}\right)^2 \frac{1}{2\mu g}$ along the plane m being the mass of the ball. **[10 Marks]**

55. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$ where $r^2 = x^2 + y^2 + z^2$, prove that the liquid motion is possible and that the velocity potential is $\cos\theta / r^2$ **[10 Marks]**

56. Prove that the equation of motion of a homogeneous Inviscid liquid moving under conservative forces may be written as $\frac{\partial \vec{q}}{\partial t} - \vec{q} \times curl\ \vec{q} = -grad\left[\frac{p}{\rho} + \frac{1}{2}q^2 + \vec{\Omega}\right]$ **[14 Marks]**